SUMMARY OF MY RESEARCH

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I have researched on the transformation group theory since my master course. One of the important problems of the transformation group theory is the classification problem. Especially my interesting problems, which are concerned with the classification problem, are smooth actions of the non-compact Lie group and the equivariant cohomology. The numbers which appear in the followings accord with the numbers of my papers in the List of my paper.

0.1. **On the classification of the compact Lie group action.** In the paper (2), I completely classified compact Lie groups, which act on the simply connected compact manifold M whose rational cohomology ring is isomorphic to the cohomology of the complex quadric with codimension one principal orbits, and the topological type of M. This is motivated by the similar research on the complex projective space $\mathbb{C}P(n)$ by F. Uchida in 1978, and in his research the odd degree complex quadric Q_{2n+1} appears except $\mathbb{C}P(n)$ as a topological type of manifolds. So I considered the even degree complex quadric Q_{2n}. I got eight actions, contain one manifold which is not the complex quadric and one group action on the complex quadric which did not get from the research of A. Kollross in 2002.

0.2. **On the smooth non-compact Lie group action.** In general a set of all compact Lie group actions on a manifold is a discrete space. However a set of all non-compact Lie group actions on a manifold is not so (a continuous space). From these points, there exists a big difference between compact Lie group actions and non-compact Lie group actions on a classification problem. Studying on non-compact group actions, we add some assumptions. For instance if we add the assumption the action is real analytic, then the research is rather easier because the action is linearizable around the fixed points. But a C^∞ action (a smooth action), which is vaster condition than the real analytic action, is difficult to study because it is not linearizable around the fixed points.

Therefore I considered examples of smooth actions for the first step, in the paper (1) ($\mathbb{H}$ is the quaternionic number) I constructed smooth actions of $SL(m,\mathbb{H}) \times SL(n,\mathbb{H})$ on $S^{4(m+n)-1}$ and I got infinitely many smooth actions.

In the paper (3) I constructed an $SL(3,\mathbb{R})$-action which is an extended action of the $SO(3)$ conjugated action on $S^4 \subset \mathfrak{sym}(3)$. This is motivated by the problem "Are there smooth $SL(3,\mathbb{R})$-actions which are extended actions of such $SO(3)$-action on S^4?" which was raised by F. Uchida in 1985. I succeeded to get an action different from his. Moreover I reproved the fact, which used in construction, that the quotient space of $\mathbb{C}P(2)$ by complex conjugation is S^4. Since the action appeared in (3) is not smooth, I have confidence that there is no such smooth $SL(3,\mathbb{R})$-action.

0.3. **On the equivariant cohomology.** Since the classification is difficult in general even for a compact Lie group action, it is natural to consider an inavariant. There is the equivariant cohomology as an invariant of the group actions. I studied in the paper (4) about this. A GKM-graph is an n-valent graph with labels (in the dual Lie algebra of torus $\mathfrak{t}^*$) on edges. This graph plays an important role to study some manifold with a torus action. If the manifold M satisfies the conditions [1] fixed points are finite, [2] pairwise linearly independent around fixed points, [3] M is an equivariantly formal space, then we get an n-valent graph (if $\dim M = 2n$), because we regard a fixed point as a vertex, the connected component of the set of one dimensional orbits S^2 which connects two fixed points as an edge and $\mathbb{C}$ which is out going from a fixed point as a leg (half line from a vertex). Moreover we attach a label by the isotropy weight representation of torus on the tangent space of each fixed point. Therefore we have the GKM-graph $\Gamma(M)$ from M which satisfies the above three conditions. We also define the equivariant graph cohomology ring $H_T^*(\Gamma)$ from one GKM-graph Γ. The important theorem is that the equivariant cohomology $H_T^*(M)$ defined by some T-manifold M is isomorphic to $H_T^*(\Gamma(M))$. Hence we can compute $H_T^*(\Gamma)$ instead of $H_T^*(M)$. But it is difficult to compute $H_T^*(\Gamma)$ in general. There is a case which we can compute it very easy. Such case is the case $H_T^*(\Gamma)$ is isomorphic to a ring defined by the combinatorial structure of the graph. For example a torus graph which contains the GKM-graphs defined by torus manifolds by Maeda, Masuda and Panov is such a case. So I defined a hypertorus graph as a new class of the GKM-graph and studied a combinatorial discription of its equivariant graph cohomology in (4). This class contains the GKM-graph defined by the hypertoric manifolds or the cotangent bundles of the torus manifold.