DEPTHS OF THE FOLIATIONS ON 3-MANIFOLDS EACH ADMITTING EXACTLY ONE DEPTH 0 LEAF

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In [C-C], Cantwell-Conlon introduced a knot invariant, called depth, with making use of the Gabai's work on the existence of finite depth foliations on exteriors of knots in S^3, and studied the invariant in a sequence of papers.

Here, we note that in their research, they often assumed that each of the foliations under consideration has exactly one depth 0 leaf. Hence naturally the next question occurs.

Question. For a given 3-manifold, is there a difference between the minimal depth of the foliations each of which has exactly one depth 0 leaf and that of the foliations each of which has more than one depth 0 leaves?

In this talk, we discuss this question for $\Sigma^{(n)}(K,0)$, the n-fold cyclic covering space of $S^3(K,0)$ where $S^3(K,0)$ denotes the manifold obtained from S^3 by performing 0-surgery on a knot K. The main result is as follows.

Theorem. Let K be a 0-twisted double of a non-cable knot.
Then we have

$$\mathrm{depth}^0_{1,[T^{(n)}]}(\Sigma^{(n)}(K,0)) \geq 1 + [\frac{n}{2}] \ ,$$

where $\mathrm{depth}^0_{1,[T^{(n)}]}$ denotes the minimal depth of C^0-foliations on $\Sigma^{(n)}(K,0)$ each of which has exactly one depth 0 leaf representing the homology class corresponding to a generator of $H_1(S^3(K,0))$.

We note that if k is the minimal depth of C^0-foliation on $S^3(K,0)$ each of which has only one depth 0 leaf, then $\Sigma^{(n)}(K,0)$ admits a depth k C^0-foliation obtained by lifting the depth k C^0-foliation on $S^3(K,0)$, which has more than one depth 0 leaves. This together with Theorem gives an affirmative answer to Question.

[C-C] J. Cantwell and L. Conlon, Topology its Applications **42** (1991) 277–289.

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