A time-independent approach to the study of spectral shift functions

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We consider a pair of self-adjoint operators $P_0 = -h^2\Delta + x_1$ and $P = P_0 + V(x)$ on the euclidean space $\mathbb{R}^n$, where h is a small parameter and $V(x)$ is a potential decaying sufficiently fast at infinity. We study the asymptotic behavior of the spectral shift function $s_h(\lambda)$ defined as distribution satisfying

$$\mathrm{tr}\,(f(P) - f(P_0)) = -\langle s_h', f\rangle, \quad \forall f \in C_0^\infty(\mathbb{R}).$$

We will show in particular that, under the non-trapping condition on the energy λ, the derivative $s_h'(\lambda)$ has a pointwise complete asymptotic expansion

$$s_h'(\lambda) \sim (2\pi h)^{-n} \sum_{j=0}^{\infty} \gamma_{2j}(\lambda) h^{2j}$$

with an explicit Weyl type formula of γ_0.

This is an analogy of the well-known result by Robert and Tamura in 1984. They studied the case without Stark effect by constructing the long time parametrix for the time evolution operator.

There are situations such as Stark case, systems etc. where such a construction is quite difficult. In this talk, we propose a time-independent approach for this problem using the Helffer-Sjöstrand formula for the functional calculus:

$$f(P) = -\frac{1}{\pi}\int_{\mathbb{C}} \overline{\partial}\tilde{f}(z)(z-P)^{-1}L(dz),$$

where $\tilde{f}(z)$ is an almost analytic extension of $f(\lambda) \in C_0^\infty(\mathbb{R})$, and $L(dz) = d(\Re z)d(\Im z)$. Roughly speaking, the non-trapping condition enables us to extend the trace of the resolvent analytically from the upper half plane to the zone $\Im z > -Mh\log\frac{1}{h}$ for any $M > 0$. This means, in the time integral expression of $s_h(\lambda)$, that the only contribution comes from a small neighborhood of $t = 0$.

This is a joint work with Mouez Dimassi (Univ. Bordeaux I).