Numerical invariants from knot groups

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Abstract. In 2006, Jiming Ma and Ruifeng Qiu defined a numerical invariant for a knot in S^3.(Chin.Ann. Math. 27) Let K be a knot in S^3, G the group of K, and G' the commutator subgroup of G. An invariant, denoted by $a(K)$, is the minimal number of elements which generate G' normally in G. They showed the following results:

Theorem [Ma-Qiu]. (1) $m(K) \leq a(K) \leq u(K)$,
(2) $\max\{a(K_1), a(K_2)\} \leq a(K_1 \sharp K_2) \leq a(K_1) + a(K_2)$, where $m(K)$ is the Nakanishi index of K, $u(K)$ is the unknotting number of K, and $K_1 \sharp K_2$ is the connected sum of knots K_1 and K_2.

We showed the following:

Main Theorem. $a(K) \leq r(K) - 1$, where $r(K)$ is the rank of K.

By combining with known results, we have:

Corollary. $m(K) \leq a(K) \leq \min\{u(K), t(K)\}$, where $t(K)$ is the tunnel number of K.

We remark that $a(K) \geq 1$ for a non-trivial knot K. We would like to use the invariant $a(K)$ for additivity problems of $u(K)$ and $t(K)$ as a lower bound.