Explicit moduli for closed genus 2 surfaces

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(Joint work with Armando Rodado)

Abstract. We explicitly describe the Teichmuller and Moduli spaces for closed surfaces of genus 2, following the path suggested by Rivin, Leibon and Springborn: (Compactified) Teichmuller space is tiled by copies of 10 explicit 6-dimensional polyhedra, each parametrizing the possible realizations in hyperbolic geometry of a Delauney triangulation/circle pattern with one of 10 specified underlying graphs. Coordinates for the polyhedra allow the surface to be explicitly reconstructed as a hyperbolic surface. Symmetries of the polyhedra can be explicitly described, thereby giving the corresponding decomposition of moduli space. This answers, in the genus 2 case, questions raised by Sullivan and Witten in recent years: that Weierstrass points may help to describe moduli for closed surfaces, and that there may be a cell decomposition with natural compactification for closed surfaces of genus 2 or more.

This is the first explicit cell decomposition of the (compactified) moduli space of any closed hyperbolic surface. The approach uses the fixed points of the unique hyperelliptic involution on genus 2 surfaces, which is an isometry with respect to any hyperbolic structure, and had been suggested by Aitchison at Xi'an in 2002. Rodado completed his PhD at Melbourne, implementing this approach using linear programming underlying circle patterns, and finding all candidate graphs describing generic Delaunay circle patterns. We thus describe Rodado's work, and subsequent joint work explicitly describing the 10 6-dimensional polytopes of the natural compactification of Teichmuller and Moduli space.