Properties of Gauss phrase and category of regions

Teruhisa Kadokami

East China Normal University

Abstract. A Gauss phrase is a totally ordered $2n$ letters in which the same letter appears twice. It is described by a map w from the ordinary ordered set $\{1, 2, ..., 2n\}$ to an unordered set $\{1, 2, ..., n\}$. We note that the order of the former set is important, and the latter set is just a labelling set (i.e. we can take the latter set like $\{dog, cat, ..., bird\}$). By dividing the former set into m parts, we obtain an m-component Gauss phrase. The case $m = 1$ is said a Gauss word. An element of the latter set is said a crossing. If we introduce the sign $+/-$ into every crossing, then we obtain a signed Gauss phrase, where the sign corresponds to the local intersection number of two arcs near the crossing from the parameter. By closing the ends, we obtain an m-component flat virtual link diagram. So we can say that a Gauss phrase is a "pre-flat virtual link diagram". By introducing over/under information to every crossing of a signed Gauss phrase, we obtain a virtual link diagram. You can introduce some other structures (for example, Reidemeister equivalence) according to your purpose.

Firstly, we study properties of a Gauss phrase. As an example, we study checkability of a diagram. The property concerns deeply with an alternating virtual link. In particular, we discuss about uniqueness of minimal genus assignment for special cases, and non-classicality of one-virtualized diagram.

Secondly, from the techniques above, we discuss about operations among the complement regions of a diagram, and we define a category of regions. The concept is a dual of nanoword theory. This kind point of view has been already applied in proving Tait's flyping conjecture and in 2-knot theory. We will apply for wider areas in the future study.