(1, 2), weak (1, 3), and strong (1, 3) homotopies on knot projections

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Abstract. The speaker plans to talk about a joint work with Yusuke Takimura (Waseda University, School of Education, M2). First, we obtain the necessary and sufficient condition that when two knot projections are related by a finite sequence of the first and second flat Reidemeister moves. Second, we introduce weak $(1, 3)$ homotopy that is an equivalence relation on knot projections, defined by the first flat Reidemeister move and one of the third flat Reidemeister moves. Third, using a map sending weak $(1, 3)$ homotopy classes to knot isotopy classes, we determine which knot projections are trivialized under weak $(1, 3)$ homotopy.

If time permits, the speaker will discuss another joint work with Y. Takimura and K. Taniyama. The joint work introduces strong $(1, 3)$ homotopy that is an equivalence relation on knot projection, defined by the first flat Reidemeister move and another type of the third flat Reidemeister moves. Showing that Hanaki's trivializing number is weak $(1, 3)$ invariant and introducing cross chord numbers that produce a strong $(1, 3)$ invariant, we claim that two knot projections having trivializing number two are weak $(1, 3)$ homotopy equivalent and strong $(1, 3)$ homotopy equivalent if and only if the two knot projections with trivializing number two can be related by only the first flat Reidemeister moves. We also determine the strong $(1, 3)$ equivalence class containing the trivial knot projection and other classes of knot projections.