Result of research

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I have studied surfaces in the 4-space via quandle cocycle invariants and surface braids. By a surface-link, we mean an oriented closed surface embedded in the 4-sphere S^4, and by a singular surface-link, we mean an oriented closed surface generic immersed in the 4-sphere S^4. A surface-link is a singular surface-link. J. S. Carter, D. Jelsovsky, S. Kamada, L. Langford and M. Saito introduced quandle homology theory, and defined invariants of surface-links, which are called quandle cocycle invariants, for each quandle 3-cocycle. The notation of surface braids was introduced by O.Viro. It is known Alexander's theorem and Markov's theorem in dimension 4, and hence, surface braids are closely related to surface-links. I also have studied spatial graphs and handlebody-links. A handlebody-link is a disjoint union of handlebodies embedded in the 3-sphere, it is represented by a spatial graph. The results of my research will be given in the following list. The reference number in the following is corresponding to the number in list of publications.

1. triple point cancelling number. It is known that attaching a finite number of 1-handles to a surface-link F is an unknotting operation and triple point cancelling operation of F by F. Hosokawa and A. Kawauchi. Thus, we can define the triple point cancelling number of F. In [3], we detect a method to bound lower of the triple point cancelling numbers of F via quandle cocycle invariants of F. It is also see that there is a surface-link F such that the triple point cancelling number of F is n for any natural number n.

2. w-index. The w-indices of a surface-link F is the minimal number of triple points of surface braids whose closures are ambient isotopic to F in S^4. I. Hasegawa proved that there is a S^2 -link whose w-index is 6. We detect a method to bound lower of the w-indices of F via quandle cocycle invariants of F ([7]). In particular, if a quandle cocycle invariant of a surface-link F associated with 3-cocycle θ_3 of the dihedral quandle of order 3 is non-trivial, then the w-index of F is at least 6. In the case it is satisfied above condition for the 3-cocycle of the dihedral quandle of order $p \neq 3$ where p is an odd prime, the w-index is 7. It is also seen that there is a surface-link F with any genus such that the w-index of F is 6.

3. calculations of quandle cocycle invariants of twist spun links. We can also define invariants of classical links for each quandle 3-cocycle. The quandle cocycle invariants of twist spin of a classical link L can be calculated by using the invariants of L for same quandle 3-cocycle. In general, we need the information of signs of all double points to calculate the invariants of L, and it is complicated. We detect to be able to calculate the invariants of L for θ_p without information of signs of all double points, and we calculated quandle cocycle invariants associated with θ_p of twist spun 2-bridge knot, twist spun pretzel links and twist spun torus links ([2,4,5]).

4. cocycle invariants of handlebody-links. A. Ishii determined Reidemeister moves for handlebody-links and defined a kei coloring of handlebody-links. He also defined invariants of handlebody-knots associated with θ_p. At present, the applicant and A. Ishii have studied quandle colorings by a finite quandle, new homology and cohomology theory and cocycle invariants. Our invariants also invariants of spatial graphs. We know that invariants are useful to consider the chirality of handlebody-links and spatial graphs and non-triviality of spatial graphs. Our colorings also distinguished two spatial graphs which has same Yamada polynomials. This research is given in [8].

5. crossing change of singular surface braids. A singular surface braid is a surface braid with transverse double points. A crossing change is an operation for a singular surface braid S inserting a pair of positive and negative crossing points along a chord that is a straight segment connecting adjacent sheets of S. In [6], we proved that crossing changes are unknotting operation of singular surface braids. By C. A. Giller, it is known that similar local moves of surface-links are unknotting operations. We also proved the Giller's theorem as a consequence of our result.

6. finite type invariants of singular surface braids. S. Kamada introduced finite type invariants of surface-links associated with crossing changes and 1-handle surgeries. In [1], we consider finite type invariants associated with 1-handle surgeries of singular surface braids. These invariants are controlled by three finite type invariants, which are double point number, Euler characteristic and normal euler number. In [6], we also consider finite type invariants associated with crossing changes of singular surface braids. These invariants are controlled by double point number and for each component, sheet number, Euler characteristic and normal euler number.

7. singular surface-links whose braid indices are at most 3. Recently, we proved that any singular surface-links whose braid indices are at most 2 are unknotted and any singular surface-links with braid index 3 are singular ribbon. A similar result was given by S. Kamada for surface-links. To obtain our result, we also proved that the w-index of a singular surface-link F is 0 if and only if F is singular ribbon. We also proved that the unknotting numbers of non-trivial singular surface-links with braid index 3 associated with 1-handle surgeries and crossing changes are 1.