# Research plan

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(0). Let $\Gamma = \begin{bmatrix} T & 0 \\ M & U \end{bmatrix}$ be a triangular matrix algebra where $T$ and $U$ are algebras and $M$ is a $U$-$T$-bimodule. Note that the derived dimension of $\Gamma$ is greater than or equal to that of $T$ and $U$ (Asadollahi-Hafezi). First, I will investigate the two questions below on the derived dimensions of several triangular matrix algebras.

**Question A**: Let $A$ be an Iwanaga-Gorenstein (=IG) algebra having selfinjective dimension $d$ and $\Gamma = \begin{bmatrix} A & 0 \\ A & A \end{bmatrix}$ the triangular matrix algebra. Note that $\Gamma$ is an IG algebra having selfinjective dimension $d+1$ (Happel). Then, is there an inequality between the derived dimensions of $A$ and $\Gamma$ with respect to the category of Cohen-Macaulay (=CM) modules ?

Namely, I would like to establish the relationship between the selfinjective dimension and the derived dimension with respect to the category of CM modules for an IG algebra because an algebra having global dimension $d$ is an IG algebra having selfinjective dimension $d$, and then the global dimension and the derived dimension with respect to the category of CM modules coincide (Krause-Kussin).

**Question B**: Let $S$ be an algebra over an algebraically closed field $k$, $M$ an $S$-module and $\Gamma = \begin{bmatrix} k & 0 \\ M & S \end{bmatrix}$ the one-point extension algebra of $S$ by $M$. Then, what is the condition when the derived dimension of $S$ and $\Gamma$ coincide ?

This question is closely related to Problem I in 'Abstract of the results of my research' because all canonical algebras are obtained as the one-point extension algebras of path algebras.

(I). An algebra is said to be *tame* if all but only finitely many isoclasses of indecomposable modules with the same dimension are controlled by only finitely many parameters. It is known that the representation type of algebras can be divided into tame and wild (Drozd). For a tame selfinjective algebra, we have the following conjecture in connection with the representation dimension.

**Conjecture**: Any tame selfinjective algebra has stable dimension at most 1.

Under a certain assumption, an upper bound of the stable dimension of the selfinjective algebra which is obtained as the orbit algebra $\hat{B}/G$ of the repetitive algebra $\hat{B}$ of an algebra $B$ having finite global dimension is given by the derived dimension of $B$ (cf. [3]). By using this fact, I verified that any standard selfinjective algebra of polynomial growth has stable dimension at most 1, and hence I obtained an evidence for the conjecture above. Moreover, applying this fact to a selfinjective algebra of wild canonical type, it turns out that an upper bound of its stable dimension is given by the derived dimension of a wild canonical algebra. Any canonical algebra has global dimension at most 2. Thus, I will second consider the question whether the derived dimension of a wild canonical algebra is 2. Question B above should be useful for this. And then I will decide the stable dimension of a selfinjective algebra of wild canonical type. In consideration of the conjecture above, I hope that the selfinjective algebra has stable dimension 2.

(II). According to Rouquier, an exterior algebra $\wedge(k^n)$ has representation dimension $n+1$, derived dimension $n$ and stable dimension $n-1$. On the other hand, any representation-finite selfinjective algebra has representation dimension 2, derived dimension 1 and stable dimension 0 (Auslander, Chen-Ye-Zhang and Han). Thus, we have a natural question.

**Question C**: For a selfinjective algebra, are the difference between the representation dimension and the derived dimension and the difference between the derived dimension and the stable dimension at least 1 respectively ?

At present, it is not known whether there exists an selfinjective algebra such that these dimensions coincide. Oppermann also pointed out it. Therefore, the third purpose is to investigate this question, and then to understand the representation theoretic properties of selfinjective algebras having low derived dimension.

(III). For an IG algebra, the category of CM modules is a Frobenius category, and then its stable category forms a triangulated category. Note that the dimension of its stable category is equal to the stable dimension. If an IG algebra is of finite CM representation type, then it has stable dimension 0. Then a natural question arises as to whether the converse should also hold. Hence, the fourth purpose is to investigate this question. This is a generalization of my result on selfinjective algebras having stable dimension 0.