Future Research Plans

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The objects of my research are"degenerations of curves" and their deformations called "splitting families". Especially, I study the topological monodromies of degenerations of curves. The topological monodromy (monodromy, for short) is an invariant of the topological type of degenerations of curves and it is expressed as a conjugacy class of the mapping class group. Since both algebro geometric and topological methods are necessary to study the monodromy, then the results contribute and influence both fields. Actually, in the case where characteristics is zero, the stable reduction theorem is easily proved from the result of the monodromy, and the classification of periodic elements of mapping class group which commute with the hyperelliptic involution are given by using some techniques of algebraic geometry([3]).

One of the purposes of my research is to obtain the results which cannot be obtained by studying from a viewpoint only algebraic geometry or topology. The aims of my research is as follows:

(i) To solve the conjecture "Any monodromy which commutes with a hyperelliptic involution is realized as the monodromy of a hyperelliptic degeneration".

(ii) To reprove Beauville conjecture (Tan's theorem) by using the theory of monodromy.

(iii) To revise Cadavid's conjecture and solve it.

(**i**) I proved the conjecture in the case where the monodromies are semistable([2]). Using this result, I hope that I can settle the conjecture affirmatively in near future.

(**ii**) The statement of Beauville conjecture (Tan's Theorem) is "Any semistable families of curves of genus $g > 1$ over $\mathbf{P}^1$ has at least five singular fibers". Tan proved this conjecture by using the method of algebraic geometry, i.e., by numerical arguments. If my idea succeeds, then we will obtain greater detail about the semistable families over $\mathbf{P}^1$, for example, about the positions of vanishing cycles, or types of singular fibers.

(**iii**) Although I constructed a counter example of Cadavid's Conjecture, the conjecture is still attractive because it is true in many cases. Thus, I want to revise this conjecture and prove it.