Result of research

Masahide Iwakiri

I have studied surface links and handlebody-links via quandle cocycle invariants and also have studied surface braids. A surface link is an oriented closed surface embedded in the 4-sphere S^4. A handlebody-link is a spatial graph, that is, a knotted graph in the 3-sphere S^3, and it represents a knotted handlebody in S^3. We introduce an equivalence relation for handlebody-links by ambient isotopies of S^3 and some move, which are called "IH-moves". J. S. Carter, D. Jelsovsky, S. Kamada, L. Langford and M. Saito introduced quandle homology theory, and defined invariants of surface links, which are called quandle cocycle invariants, for each quandle 3-cocyle. The notation of surface braids was introduced by O.Viro. It is known Alexander's theorem and Markov's theorem in dimension 4, and hence, surface braids are closely related to surface links. The results of my research will be given in the following list. The reference number in the following is corresponding to the number in list of publications.

1. triple point cancelling number. It is known that attaching a finite number of 1-handles to a surface link F is an unknotting operation and triple point cancelling operation of F by F. Hosokawa and A. Kawauchi. Thus, we can define the triple point cancelling number of F. In [3], we detect a method to bound lower of the triple point cancelling numbers of F via quandle cocycle invariants of F. It is also see that there is a surface link F such that the triple point cancelling number of F is n for any natural number n.

2. w-index. The w-indices of a surface link F is the minimal number of triple points of surface braids whose closures are ambient isotopic to F in S^4. I. Hasegawa proved that there is a S^2 -link whose w-index is 6. We detect a method to bound lower of the w-indices of F via quandle cocycle invariants of F ([7]). In particular, if a quandle cocycle invariant of a surface link F associated with 3-cocycle θ_3 of the dihedral quandle of order 3 is non-trivial, then the w-index of F is at least 6. In the case it is satisfied above condition for the 3-cocycle of the dihedral quandle of order $p \neq 3$ where p is an odd prime, the w-index is 7. It is also seen that there is a surface link F with any genus such that the w-index of F is 6.

3. calculations of quandle cocycle invariants of twist spun links. We can also define invariants of classical links for each quandle 3-cocycle. The quandle cocycle invariants of twist spin of a classical link L can be calculated by using the invariants of L for same quandle 3-cocycle. In general, we need the information of signs of all double points to calculate the invariants of L, and it is complicated. We detect to be able to calculate the invariants of L for θ_p without information of signs of all double points, and we calculated quandle cocycle invariants associated with θ_p of twist spun 2-bridge knot([2]) .

In [4], we calculated quandle cocycle invariants of twist spun pretzel links. In this case, we need essentially to calculate in $\mathbf{Z}/p^2\mathbf{Z}$. This is the main content of my doctor thesis.

In [5], we calculated quandle cocycle invariants of twist spun torus links. This result is extension of S. Asami and S. Satoh's result for twist spun torus knots.

4. cocycle invariants of handlebody-links. In last year, A. Ishii determined Reidemeister moves for handlebody-links and defined a kei coloring of handlebody-links. He also defined invariants of handlebody-knots associated with θ_p. At present, the applicant and A. Ishii have studied quandle colorings by a finite quandle, new homology and cohomology theory and cocycle invariants. Our invariants also invariants of spatial graphs. We know that invariants are useful to consider the chirality of handlebody-links and spatial graphs and non-triviality of spatial graphs. This research is given in [17].

5. crossing change of singular surface braids. A singular surface braid is a surface braid with transverse double points. A crossing change is an operation for a singular surface braid S inserting a pair of positive and negative crossing points along a chord that is a straight segment connecting adjacent sheets of S. In [6], we proved that crossing changes are unknotting operation of singular surface braids. By C. A. Giller, it is known that similar local moves of surface links are unknotting operations. We also proved the Giller's theorem as a consequence of our result.

6. finite type invariants of singular surface braids. S. Kamada introduced finite type invariants of surface links associated with crossing changes and 1-handle surgeries. In [1], we consider finite type invariants associated with 1-handle surgeries of singular surface braids. These invariants are controlled by three finite type invariants, which are double point number, Euler characteristic and normal euler number. In [6], we also consider finite type invariants associated with crossing changes of singular surface braids. These invariants are controlled by double point number and for each component, sheet number, Euler characteristic and normal euler number.