Reserch Plan

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[Reserch Task1] A generalization of the spin mapping class group into the case of a non-orientable surface

In general, some orientable manifolds with its tangent bundle having the trivial second Stiefel-Whitney class admit finite spin structures. Especially, for any orientable manifold whose dimension is less than or equal to three, including orientable surfaces, always admits a spin structure. Not every diffeomorphism preserves the given spin structure, however, the isotopies classes which preserve the given spin structure form the subgroup of the mapping class group of the surface, which is commonly called the spin mapping class group. I am thinking of the non-orientable version of spin mapping class group, that is, the subgroup of the mapping class group of a non-orientable surface consisting of isotopy classes which keep pin structures, which are counterparts of spin structures defined on certain non-orientable surfaces. I conjecture that the pin mapping class group can be characterized clearly as a subgroup of the mapping class group of a non-orientable surface. For example, the subgroup can be conjectured to coincide with the group consisting of elements extendable to a self-diffeomorphism of some 4-manifold as a subgroup of the mapping class group of a non-orientable surface. To show this, I would like to reserch on the mapping class group of a non-orientable surface and quadratic forms and its generalizations, which are counterparts of the notion of pin structures.

[Reserch Task2] The determination of the spin-preserving symplectic groups and its applications

As described in the paper list[2], [3], the presentation of spin-preserving symplectic groups, which is the homological version of the spin mapping class group, was determined explicitly only in the case of the genus of the surface is less than or equal to two. I have to determine the simple presentation of this group in the case of the surface of any genus and in the case of the non-orientable surface-knots, which are relevant to the reserch task2 described above. First of all, in the case of orientable surface-knots, for the relatively well-known surface-knots, for example, twisted-spun knots and ribbon knots, I have to compute the spin-preserving symplectic group. Then I wish to characterize the family of surface-knots classified by this invariants, which investigates the strength as an invariant as the next task. The study of these problems in the case of non-orientable surfaces on pin-preserving symplectic group should be dealt with after the research plan 1 mentioned above is fulfilled.