RESEARCH PLAN

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In the study of Riemannian symmetric spaces, it is important to find homologically volume minimizing submanifolds. Since volume minimizing totally geodesic submanifolds are stable totally geodesic submanifolds, they are interesting objects in terms of geometric variational problem.

On the other hand, a symmetric R-space is defined by a maximal totally real, totally geodesic submanifold of a certain Hermitian symmetric space. Because symmetric R-spaces have special structures, it is a fundamental problem to characterize their property.

To this context, my research plan is as follows.

1. Riemannian symmetric spaces and calibrated geometry

R. Harvey and H. B. Lawson initiated calibrated geometry in order to study currents with minimal mass. From the point of view that calibrated submanifolds are stable minimal submanifolds, calibrated geometry is related with the theory of minimal submanifolds. It is known that Helgason spheres in certain compact symmetric spaces, Hermitian subspaces in Hermitian symmetric spaces and quaternionic Kähler subspaces in quaternionic Kähler symmetric spaces are calibrated submanifolds. We try to construct calibrations which calibrate totally geodesic submanifolds in compact Riemannian symmetric spaces of rank two, which were classified in [2]. We also try to construct calibrations which calibrate reflective submanifolds defined by certain cohomogeneity one actions.

2. symmetric R-space

From the point of view of algebra, symmetric R-spaces are related with graded Lie algebra of the first kind. On the other hand, twistor spaces over symmetric R-spaces are obtained by graded Lie algebra. To this context, we investigate twistor spaces over symmetric R-spaces and try to construct certain kind of homogeneous spaces associated symmetric R-spaces from graded Lie algebra under consideration of S. Kobayashi–T. Nagano's result. Furthermore we study common geometric structures among these homogeneous spaces.

3. Integral geometry and volume minimizing

We consider that the Poincaré formula in Riemannian homogeneous spaces given by R. Howard is a necessary to determine the classes of volume minimizing submanifolds. First, we remember that certain submanifolds of a real space form and a complex space form satisfy the Poincaré formula. Next, we will show by applying the Poincaré formula that stable totally geodesic submanifolds as an object which were obtained by [2] and [3] are volume minimizing.