Study plan.

[1] Phase diagram of the $q\bar{q}$ and qq condensates in the 3-dimensional Gross Neveu model with 4 component spinor representation for quarks

It is one of the most exciting subject to study the phase structure of Quantum Chromodynamics (QCD). QCD is an asymptotically free theory and the interactions between quarks and gluons become small at high energy. Then quarks and gluons are free particles at high temperature and density and this state is called Quark Gluon Plasma (QGP). On the other hand at low temperature and density, quarks and gluons are confined within hadrons. There are thus two distinct phases, the "hadronic phase" where quarks and gluons are confined, and QGP phase where they are deconfined. This remained the standard picture of the QCD phase diagram in 1970's and 1980's.

However, the possibility of the new state "color superconductivity" was pointed out in 1975 and it was widely accepted in the end of the 1990s. Color superconductivity is the state where the quark-quark Cooper pairs are induced by the attractive interaction in the color antitriplet channel. In 1998, it was revealed that the related gaps in the fermion spectrum could be of the order of 100MeV, much larger than expected earlier. This means the color superconducting phase has non-negligible region in the QCD phase diagram. Then it is now believed that various color superconducting phases are realized at low temperature and relatively high baryon density.

The above mentioned aspect of the QCD phase diagram was successfully described by the Nambu Jona-Lasinio (NJL) model which is a low energy effective field theory of QCD. The NJL model has the same symmetry as QCD and it had been studied in detail.

In the paper "Phase diagram of quark-antiquark...", I have focused on the Gross Neveu (GN) model with, the counterpart of the NJL model in lower dimensions (D), and study the phase structure of the model in 3D. In 3D, the GN model is investigated by employing the 2 component (2c) and 4 component (4c) spinor representation for quarks, and I have studied the 2c case. It is possible to perform the same analysis in the 4c case by following the same procedure that I have already done. Then I am studying the phase diagram of the $q\bar{q}$ and qq condensates in the 3-dimensional Gross Neveu model with 4c spinor representation for quarks.

The plan of this research is as follows. Firstly we employ the Lagrangian density of the 3D GN model with 4c representation, and apply the mean-field approximation. Then we evaluate the thermodynamic potential through computing the partition function of the model. The thermodynamic potential includes the ultraviolet divergences as in the standard GN model, and we perform the renormalization to eliminate the divergences. Finally by seeking the minimum of the thermodynamic potential with respect to the order parameters for the $q\bar{q}$ and qq condensates, we construct the phase diagram of the model.

[2] Phase diagram of $q\bar{q}$ and qq condensates in the 2D Gross Neveu model

The study of the phase structure of $q\bar{q}$ and qq condensates in the 2D GN model is also an interesting issue. Following the same manner as explained above, it is possible to construct the phase diagram in 2D GN model. Then I am studying the phase diagram in the 2D GN model after obtaining the results in 3D GN model with the 4c representation. With the results of the phase structures, we can make a comparison among the four-fermions interacting field theory in various dimensions, namely the 2D and 3D GN model and 4D NJL model, which is interesting from theoretical and phenomenological point of views.