Results of our study

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There are many early results and interesting examples for complex submanifolds, totally real submanifolds, *CR* submanifolds and real hypersurfaces of a complex space form.

However, the study of general submanifolds in a complex space form has some difficulties. For example, fundamental geometrical conditions such as `totally geodesic' do not work well. It is known that if a submanifold in a complex space form is totally geodesic, then the submanifold is complex or totally real. So many authors use the Hopf fibration to study submanifolds in a complex projective space. To solve the difficulty, we computed the Simons type integral formula without using the Hopf fibration, and gave some pinching theorems for general real submanifolds in a complex space form ([4], [5]).

We computed the Laplacian for the square of the length of the second fundamental form of a submanifold in a complex space form of nonnegative holomorphic sectional curvature under the condition that the mean curvature vector field is parallel. Combining this with formulas for some matrixes, we gave a useful representation of Simons type integral formula. Using this formula, we proved pinching theorems with respect to the scalar curvature of a compact minimal submanifold in a complex space form of nonnegative holomorphic sectional curvature. We can characterize real hypersurfaces of type A by this pinching result ([6]).

On the other hand, we proved pinching theorems with respect to the sectional curvature. If the sectional curvature of an n-dimensional compact minimal *CR* submanifold M in complex projective space with flat normal connection is equal or greater than $1/n$, then M is congruent to the minimal geodesic hypersphere ([4]).

Moreover, we characterized compact minimal *CR* submanifolds in a complex projective space whose Ricci curvature satisfies some inequalities ([5]).

These results extend some known pinching theorems for minimal submanifolds in a complex projective space.

For the study of real hypersurface in a complex space form, we studied the second fundamental form and the Ricci tensor with respect to the condition on the holomorphic distribution.

There are no totally umbilical real hypersurface in a complex space form, so the notion of `totally η-umbilical' has been considered. We studied the condition that the second fundamental form satisfies the totally umbilical condition on the holomorphic distribution. This is the extension of the totally η-umbilical condition ([3]). On the other hand, we proved that if the Ricci tensor of a real hypersurface of a complex space form satisfies the semi-symmetric condition on the holomorphic distribution, then the real hypersurface is pseudo-Einstein ([2]). Furthermore, we classified real hypersurfaces in a complex space form with constant principal curvatures when the Ricci tensor satisfies the Einstein condition on the holomorphic distribution ([7]).