Research plan

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The theme of this research is to study Lagrangian submanifolds, Lagrangian fibrations and related topics. The relation between theta functions and certain Lagrangian fibrations on Abelian varieties can be interpreted in terms of geometric quantization or mirror symmetry. The main subjects of this research in a few years are Lagrangian fibrations of flag manifolds and K3 surfaces, which are also important examples in geometric quantization and mirror symmetry. We study these Lagrangian fibrations via degenerations of flag manifolds and K3 surfaces.

The Gelfand-Cetlin system on flag manifolds. A flag manifold $U(n)/T$ is related to a certain convex polytope, called the Gelfand-Cetlin polytope, in three different theories: the Gelfand-Cetlin system, a completely integrable system; the Gelfand-Cetlin basis, a basis of an irreducible representation of $U(n)$; and a toric degeneration. The first two relations are quite similar to the case of toric varieties. One of our purpose is to clarify the relationship between the Gelfand-Cetlin basis and the Gelfand-Cetlin system more explicitly. We also study differential geometric properties of the Gelfand-Cetlin system, such as minimality and Hamiltonian minimality of the Lagrangian fibers. The result on the toric degeneration of the Gelfand-Cetlin system will be used in this research. We study the relation from the geometric and representation theoretic view points.

We are also considering an application of the result on the toric degeneration to the study of mirror symmetry for flag manifolds. This part is a joint work with T. Nishinou and K. Ueda. For toric varieties, Cho-Oh calculated Floer cohomologies of Lagrangian torus fiber of the moment map. We would like to extend this result to the flag case using the degeneration of Gelfand-Cetlin system.

Special Lagrangian fibrations of K3 surfaces. Special Lagrangian submanifolds are examples of volume minimizing submanifolds, and hence important in the minimal submanifold theory. Moreover, special Lagrangian fibrations play crucial roles in geometric mirror symmetry proposed by Strominger-Yau-Zaslow. We study special Lagrangian fibrations of K3 surfaces and relation to other structure such as the Bergman kernel, via certain degenerations of the K3 surfaces. The study of special Lagrangian submanifolds involves the analysis of Ricci-flat Kähler metrics, which is also a difficult analytic problem.

We mainly consider two classes of K3 surfaces . The first one is a certain smoothing of singular Kummer surfaces $A/(-1)$. The degeneration in this case is "mild", and can be applied the analysis of Ricci-flat metrics by R. Kobayashi. We would like to generalize the approximation result for the singular Kummer surfaces to this case. The other is the most general case where the special Lagrangian fibration has 24 singular fibers. In this case, we will use the analysis by Gross-Wilson for the behavior of Ricci-flat metrics under a certain degeneration (a large complex structure limit). Since the K3 surfaces converge to non-compact manifolds under this degeneration, we need to find an appropriate framework of the analysis.