Plan of my research

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I approach the problem related to the classification of braids from the viewpoint of the knot theory. I plan the following researches:

(1) On an infinite sequence of mutually non-conjugate braids which close to the same knot

By the Classification Theorem of closed 3-braids given by J. S. Birman-W. W. Menasco, it is known that any link which is a closed n-braid ($n = 1, 2$ or 3) has only a finite number of conjugacy classes of n-braid representatives in the n-braid group B_n. When $n \geq 4$, the situation changes. H. R. Morton and T. Fiedler gave infinite sequences of 4-braids which result in the unknot and E. Fukunaga gave such a sequence for $(2, p)$-torus links. However, which knots (or links) have such sequences is not completely known. In [4], [6] and [7], I tried to construct such infinite sequences and gave some sequences. Morton, Fiedler and Fukunaga used conjugacy invariants which are calculated from braid words. On the other hand, to show that braids in our sequence are mutually non-conjugate I used knot invariants. Since this method doesn't work for links, I would like to improve it so that it can be used for links. In [7], generalizing sequences given by Morton, Fiedler and Fukunaga, for any knot K represented as a closed n-braid ($n \geq 3$), I gave an infinite sequence of mutually non-conjugate $(n + 1)$-braids representing K. In [4], for knots which is a closed n-braid satisfying certain conditions, I gave an infinite sequence of mutually non-conjugate n-braids representing the knot and gave some infinite sequences. Recently, using a covering space, Y. Uchida gave another proof to the result in [7]. I also would like to extend the sequences given in [4] by using his method. Birman-Menasco proved that if an oriented link has infinitely many conjugacy classes of n-braid representatives in the n-braid group, the infinitely many conjugacy classes divide into finitely many equivalence classes by exchange moves. By examining the influence of exchange moves on conjugacy invariants of braids, I would like to find the clue of the classification by exchange moves.

(2) On irreducibility of braids

H. R. Morton gave the first example of irreducible 4-braid which closes to the unknot. Later, from Morton's example, T. Fiedler constructed an infinite sequence of mutually non-conjugate irreducible 4-braids which close to the unknot. However, in general, it is difficult to show the irreducibility of braids. Hence Morton's and Fiedler's arguments in the proof of irrecucibillity of braids remains restricted to 4-braids, and the generalizations to higher braid groups has not been known yet. In [4], for a knot of even braid index $n \geq 4$ satisfying a certain conditions, I gave an infinite sequence of mutually non-conjugate n-braids which close to the knot. Since all braids in my sequence realize the braid index of the knot, they are irreducible. In the point that they are irreducible, it can be said that the sequence is a kind of extension of Fiedler's sequence. However Morton's and Fiedler's proofs of the irreducibility are different from mine. To understand the influence of the stabilizations and destabilizations on the conjugacy classes in the braid group, it seems that to examine irreducible braids is effective. Since an effective method to detect irreducibility has not been found yet, I would like to find it from the viewpoint of knot theory.