Results of my research

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(1) Homology classification of spatial graphs

K. Taniyama showed that two spatial embeddings are spatial-graph-homologous if and only if they have the same Wu invariant. In [1], we showed that two spatial embeddings of a graph are spatial-graph-homologous if and only if all of their linking numbers and Simon invariants coincide. Therefore Homology classification came to be given by the simple calculation, since both linking number and Simon invariant are integral invariants that are easily calculated from a regular diagram of a spatial graph. T. Motohashi-Taniyama showed that two spatial embeddings are spatial-graph-homologous if and only if they are delta-equivalence and Taniyama-A.Yasuhara showed that a delta-move does not change any order 1 finite type invariant of spatial graphs. Therefore it follows that linking number and Simon invariant determine all of order 1 finite type invariants of spatial graphs.

(2) Spanning surfaces of spatial graphs

A locally unknotted spatial embedding of a graph was introduced by K. Kobayashi. In [3], I got an idea from his concept and defined a collection of spanning surfaces of a spatial graph. A spatial embedding of graph G is said to be locally unknotted, if there is a set of cycles Γ of G which forms a basis for $H_1(G;\mathbb{Z})$ and the set of knots in the spatial embedding corresponding to Γ bounds a set of disks with disjoint interiors. We considered a set of connected, compact and orientable surfaces with disjoint interiors bounded by knots in a spatial graph. If each surface has a distinct boundary, we say that the set is a collection of spanning surfaces. Especially if each surface is homeomorphic to a disk, the set is called a collection of spanning disks. T. Endo-T. Otsuki proved that any graph has a locally unknotted spatial embedding. However, in general, the rank of $H_1(G;\mathbb{Z})$ is not an upper bound of the number of spanning surfaces. Hence I gave the upper bound of the number of spanning surfaces and showed that this upper bound is the least upper bound by constructing a spatial embedding which realizes the upper bound with disks.

In [2], we tried to extend the concept of boundary links to spatial graphs. We defined a boundary spatial embedding of a graph G as a spatial embedding of G with a collection of spanning surfaces which respects to the set of all knots in the embedding. From the result in [3], we have that not every graph has a boundary spatial embedding. Hence we gave a characterization of graphs which have boundary spatial embeddings. Then we classified boundary spatial embeddings of a graph completely up to self pass-equivalence and showed that any two boundary spatial embeddings of a graph are self sharp-equivalent. These are natural extensions of the results concerning boundary links given by L. Cervantes-R. A. Fenn and T. Shibuya.

(3)An infinite sequence of non-conjugate braids whose closures result in the same knot

By the Classification Theorem of closed 3-braids given by J. S. Birman-W. W. Menasco, it is known that any link which is a closed n-braid ($n = 1, 2$ or 3) has at most three conjugacy classes of n-braid representatives in the n-braid group. However $n \geq 4$, the situation changes. An infinite sequence of mutually non-conjugate 4-braids representing the unknot (resp. $(2, p)$-torus link) has been discovered. In [7], for any knot K (or a link satisfying certain conditions) represented as a closed n-braid ($n \geq 3$), I gave an infinite sequence of mutually non-conjugate $(n + 1)$-braids representing K. Recently by using braid permutations, for an n-braid which satisfies certain conditions I gave an infinite sequence of mutually non-conjugate n-braids which has the same closure as the braid.