Summary of my research

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Every oriented closed 3-manifold admits a fibered knot/link and one can decompose the manifold into a tubular neighbourhood of the knot/link and a surface bundle over S^1. This decomposition is called an open book decomposition of the given 3-manifold (an "open book" for short). Many results have been obtained with a viewpoint that a structure of a fiber surface with respect to the Murasugi sum describes the structure of the open book.

I studied the structure of fiber surfaces in S^3 with respect to Murasugi sum and obtained the following results.

- "Hopf band" is a fiber surface in S^3 with first Betti number 1, which is the simplest fiber surface except 2-disk in S^3. We may say that a "Hopf plumbing" i.e., a fiber surface which can be decomposed into some Hopf bands with respect to plumbing (or 4-Murasugi sum), has a basic strucuture of fiber surface. It is known that a fiber surface of a fibered alternating link is a Hopf plumbing.

  In a joint work with Goda at Tokyo A & T and Hirasawa at Gakushuin Univ., we proved the following: "Let R be a Seifert surface obtained by applying Seifert's algorithm to an "almost" alternating diagram. Then R is a fiber surface if and only if R is a Hopf plumbing."

- Harer showed that all fiber surfaces in S^3 can be constructed from the 2-disk by the following three operations: (1) plumbing Hopf bands, (2) deplumbing Hopf bands, and (3) Stallings twists. Stallings twist is a Dehn twist along a special simple closed curve, called a twisting loop, on the fiber surface. He then conjectured that the Stallings twist can be omitted.

  I defined a certain kind of complexity (positive integers) of Stallings twist and proved that a Stallings twist with complexity 1 is realized by plumbing and deplumbing Hopf bands.

For open books of general closed oriented 3-manifolds, I have had the followings:

- In a joint work with Toshio Saito at Nara Women's Univ., we introduced the notion called "translation distance" of open books, which is defined as the minimal length of translation of the vertices in arc complex of the fiber surface of the open book under the action of the monodromy map on the arc complex. Then we proved that:

  "The translation distances of open books are bounded above by 2 if they have a Murasugi summand and by 3 if they have a twisting loop".

- For a fibered knot K and its fiber surface Σ in a closed orientable 3-manifold M, we focus on algebraic intersection numbers of essential arcs on Σ with their images of the monodromy map of K and investigated how Alexander polynomial of the fibered knot is calculated using these information. We then showed that each coefficient of Alexander polynomial of the knot may be described by Pfaffians of some matrices of the algebraic intersection numbers.

Plumbing a positive Hopf band to an open book of a closed oriented 3-manifold M, one can obtain a new open book of M. This operation is called a positive stabilization of open books. Giroux showed a one-to-one correspondence between all equivalence classes of open books of M up to positive stabilization and all isotopy classes of positive contact structures of M.

- I studied a characterisation of open books corresponding to overtwisted contact structures through Giroux's one-to-one correspondence, and have found an important role of the twisting loop as the following:

  "An open book corresponds to an overtwisted contact structure if and only if it is equivalent up to positive stabilization to an open book with a twisting loop."