Results of research

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Papers are referred by numbers in “List of my papers”

I have been studying “the knots contained in spatial embeddings of graphs”. A graph G is a pair of a set of vertices and that of edges which join vertices. A spatial graph is the image of an embedding of the graph G in the 3-dimensional Euclidean space. The image of an embedding a 1-dimensional cycle in the 3-dimensional Euclidean space is a knot. A graph that any two vertices are joined by an edge is called the complete graph. If there exists a cycle of a spatial graph which is equivalent to a knot K, then the knot K is contained in the spatial graph. Conway-Gordon proved that every spatial embedding of the complete graph with 7 vertices contains the nontrivial knot. The purpose of my research is to study the generalization of Conway-Gordon's result.

A linear embedding of a graph is an embedding which maps each edge to a single straight line segment. A linear embedding such that each vertex of K_n is mapped into a spiral $H := \{(\cos\theta, \sin\theta, \theta) \in \mathbb{R}^3 | 0 \leq \theta \leq 2\pi\}$ is called by a column embedding. In the joint paper [1] with T. Tanaka, we proved that “every column embedding of the complete graph with $(2n-1)$ vertices or $2n$ vertices contains the torus knot of type $(2n-5, 2)$”. For column embeddings, we gave a sufficient condition for knots to be contained in it of the complete graph.

Moreover, “For a column embedding of the complete graph with 9 vertices, the spatial graph dealt with one crossing change at a vertex that appears on the xy-plane contains a knot whose crossing number is greater than or equal to 5”.

A circular embedding of a graph is defined as an embedding such that there exists an arc contained in the image of any edge. For any graph, there exist a circular embedding representing it. For a particular spatial graph, there exists an embedding which is not equivalent to it. From here, we focus on knots, which are considered as knots, which are considered as spatial graphs homeomorphic to the circle. For a knot K, the circular number $Circ(K)$ of K is defined by the minimal number of the number of vertices a circular embedding of a graph equivalent to K. In the case of linear embeddings, the number of edges consisting a nontrivial knot is at least 6. In the case of circular embeddings, the number of edges consisting a nontrivial knot is at least 3. It seems that a circular embeddings are more suitable than linear embeddings for the study of generic embeddings. In the joint paper [1] with Dr. Tanaka, we obtained inequalities of the circular number of K and some invariants such as a stick number, an arc index, a crossing number, a bridge number and a superbridge number. Moreover, we prove the following results. “A knot K is a trefoil knot if and only if $\mathrm{Circ}(K)$ is equal to 3. ” “If K is the figure-eight knot, then $\mathrm{Circ}(K)$ is equal to 4. ” “If K is the connected sum of a trefoil knot and its mirror image, then $\mathrm{Circ}(K)$ is equal to 4. ” In general, the circular number is not additive under connected sum. Finally, we defined another invariant of a knot associated with a circular embedding. Let K be a knot. We define $u_m(K)$ by the minimal number of crossing changes such that K is transformed into a circular embedding consisting of m round arcs. We obtain the following result. “For a trefoil knot K, $u_m(K)$ is 1 if $m \leq 2$ and 0 if $m \geq 3$. ” Note that if $m \leq 2$ then $u_m(K)$ is exactly equal to the unknotting number of K. So, we may say that $u_m(K)$ is a generalization of the unknotting number.