Result

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Estimation of the number of holomorphic section

Let (M, π, B) be a holomorphic family of Riemann surfaces. A holomorphic mapping $s : B \to M$ is said to be a *holomorphic section* of (M, π, B) if the composed mapping $\pi \circ s$ is the identity mapping of B.

For a given holomorphic family (M, π, B), it is a fundamental problem to estimate the number of its holomorphic sections. It is known that if the genus of the fiber $S_b = \pi^{-1}(b)$ over each point $b \in B$ is greater than or equal to 2, and if M is locally non-trivial, then the number of holomorphic sections is finite. This claim was proved by Manin and Grauert independently. Next we shall estimate the number of holomorphic sections.

In [2] of the list, we consider a family whose base space B is a torus with four punctures and, the fiber $S_b = \pi^{-1}(b), b \in B$, is a closed Riemann surface of genus two which is a two-sheeted branched covering surface of a torus branched over two points, and which is constructed by Riera. We have the following result:

In general, the triple (M, π, B) has at most four holomorphic sections.

This study can be regarded as the first step to the evaluation of the number of holomorphic sections.