Symplectic potentials are determined for a wide class of toric Sasaki-Einstein manifolds. The spectrum of the scalar Laplacian on $L^{a,b,c}$ metrics are investigated. The eigenvalue problem leads to two Heun's differential equations. The ground states and first excited states are studied in detail. It turns out that the scaling dimensions of the holomorphic functions which correspond to the ground states are consistent with R-charges of the dual quiver gauge theories ([15] of Publication List).

We present an explicit non-singular complete toric Calabi-Yau metric using the local solution recently found by Chen, Lü and Pope. This metric gives a new supergravity solution representing D3-branes ([16]).

We construct a new infinite family of quiver gauge theories which blow down to the $X^{p,q}$ quiver gauge theories. This family includes a quiver gauge theory for the third del Pezzo surface. These theories generically have irrational R-charges ([17]).

The κ -symmetry-fixed Green-Schwarz action in the $AdS_5 \times S^5$ background is treated canonically in a version of the light-cone gauge ([18]). The action is written in terms of the phase space variables. We convert it into the standard action written in terms of the fields and their derivatives. We obtain a Nambu-Goto type action which has the correct flat space limit ([23]).

We explicitly calculate the Riemannian curvature of D-dimensional metrics recently discussed by Chen, Lü and Pope. We find that they can be concisely written by using a single function. It is shown that the metrics are of type D ([19]).

Assuming the existence of a single rank-2 closed conformal Killing-Yano tensor with a certain symmetry we show that there exist mutually commuting rank-2 Killing tensors and Killing vectors. We also discuss the condition of separation of variables for the geodesic Hamilton-Jacobi equations ([20]).

We classify all spacetimes with a closed rank-2 conformal Killing-Yano (CKY) tensor. It is shown that the *D*-dimensional Kerr-NUT-de Sitter spacetime constructed by Chen-Lü-Pope is the only spacetime admitting a non-degenerate closed CKY tensor ([21]). For spacetimes with general closed CKY tensor, they give a generalization of Kerr-NUT-de Sitter (KNdS) spacetimes. The metric of the generalized KNdS spacetime is a metric of the fiber space whose base spaces are several Kähler spaces and at most one general space, and its fiber is the KNdS space ([24, 25]).

It is shown that the Dirac equations in general higher dimensional Kerr-NUT-de Sitter spacetimes are separated into ordinary differential equations ([22]).

In the genearalized Kerr-NUT-de Sitter spacetime, it is shown that certain type of tensor perturbation admits separation of variables and the linearlized Einstein equation (the Lichnerowicz equation) turns into ordinary differential equations ([26]).