Research plan

Tae Hattori

As can be seen in "Results of research", the Royden p-boundaries of two quasi isometric graphs are homeomorphic. When $p = 2$, it is known that there exist Borel measures on the Royden boundary which are called the harmonic measures. I have a conjecture that two harmonic measures on Royden boundaries of two quasi isometric graphs correspond absolutely continuously each other. As a first step to solve this conjecture, I want to compare the capacity of a subset of the boundary and the extremal length of a family of paths. It has been already known that the positivity of the capacity is invariant under the quasi isometry, but it is not clear whether the capacities have been able to be compared quantitatively between two quasi isometric graphs. The extremal length is defined for a family of paths. However, the image of a path by a quasi isometry does not always become a path. Thus it is necessary to extend the notion of extremal length.

Now we are concerned with compactifications of a graph relative to certain spaces of functions with finite p-Dirichlet sums. In the case when p is greater than 2, the Kuramochi p-compactification is also defined and there is a canonical projection from the Royden p-compactification onto the Kuramochi compactification. Moreover, when p is equal to 2, the Kuramochi boundary of a graph coincides with the Royden boundary if the effective resistance is uniformly bounded. We note that Kuramochi compactifications are always metrizable. Accordingly, we shall consider a mapping with finite p-energy from the graph to the Kuramochi p-compactification considering as a metric space. For the case when p is equal to 2, the theory of Dirichlet spaces is applicable to investigate the behavior of the mapping with finite energy in the Kuramochi compactification. But, for general p except 2, there is no such a theory at all. Therefore we need a new idea to overcome this situation.

I am also interested in the following topics relevant to my former research.

Associated to a measure on the set of vertices of a graph, we have the Laplacian acting on functions. First, we observe two canonical Laplacians. One is determined by putting the degree of each vertex as the measure on it, and there are many results on this Laplacian in relation with the random walks or as discrete analogues of Riemannian manifolds. The other one is determined by the counting measure. It has just started to study the latter. For example, the Brooks inequality is established only for the former Laplacian. For an infinite rapidly branching tree, it is known that the essential spectrum of the former Laplacian is only 1 and the spectrum of the latter Laplacian is discrete. This result is concerning the Laplacians with respect to measures mentioned above. I would like to find possibly a measure such that the spectrum of the Laplacian will be reflected to the structure of a graph.

A class of harmonic morphisms is an interesting subclass of harmonic mappings. Some comparison results on the notion of harmonic morphisms between graphs is introduced by Hajime Urakawa, and the Green functions and the spectral gaps between the domain and the target of a harmonic morphism are discussed. I am expecting that a harmonic morphism gives rise to contraction with respect to resistance metrics of the domain and the target graph.