Result of research

Kengo Kishimoto

1.Braiding and braid index: Lambropoulou-Rourke showed that every link with a fixed closed braid can be deformed into a closed braid by a deformation keeping the sublink fixed. To estimate the braid index of such a link, I gave an alternative proof by giving a diagrammatic deformation. In general, a link type is changed by reversing orientation of its sublink. It is natural to ask "how many is the difference of their braid indices?" As an application of my result, we gave an upper bound for such difference.

2.Alternativity of closed 3-braids: The dealternating number (introduced by Adams et. al.) and the alternation number (introduced by Kawauchi) are knot invariants which estimate alternativity of knots. In general, it is hard to determine these invariants for knots. Abe and I gave an upper bound for these invariants through the braid presentation. By combining Abe's lower bound for the alternation number, we detected these invariants for almost all closed positive 3-braid knots. As an application, for any non-negative integers n, m, we construct infinitely many knots with the dealternating numbers (the alternation numbers) n and the braid indices m.

3.IH-complex of spatial trivalent graphs: An IH-move is a local spatial move on spatial trivalent graphs, introduced by Ishii. Ishii and I define the IH-distance between two spatial trivalent graphs as the minimal number of IH-moves needed to deform one into the other. By extending the definition of the IH-distance, we define the IH-complex on the set of spatial trivalent graphs. We showed that the IH-complex is not locally finite, and its dimension is greater than or equal to two. By using the quandle coloring introduced by Ishii and Iwakiri, we showed that for any non-negative integer n, there exists a pair of spatial θ-curves with IH-distance n.

4.Positive knots of genus two: Stoimenow showed that positive knots of genus two are represented by the diagrams obtained from the 24 diagrams by applying $\overline{t_2'}$ moves. Jong and I showed that positive knots of genus two are positive-alternating or almost positive-alternating, where an almost positive-alternating knot is a non-positive-alternating knot which has a diagram such that a single crossing change turns the diagram into positive-alternating one. As applications, positive knots of genus two are represented by the diagrams obtained from the "14" diagrams by applying $\overline{t_2'}$ moves, and positive knots of genus two are quasi-alternating. Here a quasi-alternating knot is a generalization of an alternating knot, introduced by Ozsvath and Szabo.

5.Table of genus two handlebody-knots: A handlebody-knot is a handlebody embedded in the 3-sphere. Ishii showed that two handlebody-knots are equivalent if and only if two spatial trivalent graphs as their spines are related by IH-moves. Moriuchi enumerated spatial θ-curves and spatial handcuff graphs with up to seven crossings by using a generalization of Conway's method. Ishii, Moriuchi, Suzuki and I listed genus two handlebody-knots with up to six crossings, and we distinguish between them except for two pairs by calculating the representations of the fundamental groups of their complements into finite groups.