Summary of my research

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For an orientation-preserving homeomorphism $\varphi : S^1 \to S^1$ on the circle which has no periodic point, there exist an irrational rotation $R_\alpha : x \mapsto x+\alpha \pmod 1$ and a factor map $F : S^1 \to S^1$ satisfying $F \circ \varphi = R_\alpha \circ F$. In addition, if φ is not topologically conjugate to an irrational rotation, then φ is called a Denjoy homeomorphism. (For more details of Denjoy homeomorphism, for example, refer to (Kouichi Yano, Dynamical System 2 (in Japanese)). Denjoy system is defined as the unique minimal subsystem of Denjoy homeomorphism, and is a Cantor system. Then the set $\{\omega \in S^1 \mid \#F^{-1}(\omega) \neq 1\}$ is a union of the orbits of at most countably many points ([Poincare]). We call the cardinality of these orbits the double orbit number, and call this set double point set. The topologically cojugate class of a Denjoy homeomorphism is determined by its rotation number and the configuration of double point set ([Markley]).

In genaral, for each Cantor system, there exists an adic model (adding machine defined on the infinite path space of Bratteli diagram) which is topologically conjugate to it. But only few adic models are concretely constructed. The dimension groups associated with Cantor system is a complete invariance under strong orbit equivalence. Moreover Denjoy systems and odometers stand a complete representation of orbit equivalence class ([T. Giordano, I. F. Putnam, C. F. Skau]). Already the dimension groups associated with Denjoy systems whose double orbit is finite have been determined except for some case ([I. Putnam, K. Schmidt, C. Skau]).

In [3], we concretely construct an adic model of the Denjoy systems of finite double orbit number. In order to do this, we use the continued fraction expansion and associated Ostrowski-type expansion which are arithmetic. By using this adic model, we can completely determine the dimension groups for Denjoy systems of finite double orbit number.

In [1], we see that an adding machine corresponds to a sequence of substitutions $(\sigma_n)_{n=1,2,\ldots}$. The composition of this sequence of substitutions $\sigma_1\sigma_2\ldots\sigma_n$ approximates the natural coding sequence with some initial infinite path. Especially, the sequence of substitutions obtained by the adic model of Denjoy system given in [3] generates an infinite (double orbit number +1)-letter sequence. This is a coding sequence obtained by partitioning the circle into (double orbit number+1) arcs. Especially, when double orbit number is 1, this is a 2-letter sequence, which is known as Sturmian sequence. So this is a natural generalization of Sturmian sequence. Moreover the subshift which is generated by this is topologically conjugate to original Denjoy system.