Plan

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I have two plan. The first is to extend the result of the Riemann geometry of Ricci solitons to a result of the Lorenz geometry. In particular, I pay attention the result of the nilpotent Lie group. Secondly, I treat Einstein metrics, Ricci solitons and cohomogeneity one metrics with respect to a G-action on a 4-manifold.

The first plan is the following. Ricci solitons was studied by Hamilton, Bryant, Ivey, Perelman and many people. However the results of Ricci solitons was the Riemannian case. The results of Ricci solitons in Lorentzian case are very few. The plan is to extend the result of the Riemann geometry of Ricci solitons to a result of the Lorenz geometry. I pay attention the result of nilpotent Lie group by Lauret. Lauret showed a nil-manifold has a Ricci soliton structure if and only if it admits a metric standard solvable extension whose corresponding standard solv-manifold is Einstein. I will study Lauret's results in the Lorentzian case. I studied Ricci soliton in the Lorentzian case and continue the study of Ricci solitons. Concrete problem are a construction of left-invariant Lorentzian Ricci soliton on Lie groups with more than four dimensional.

Next, the second plan is the following. There exists a left-invariant coframe $\{\theta^i\}_{i=1}^3$ on three-dimensional unimodular Lie group G satisfying $d\theta^i = 2\theta^j \wedge \theta^k$, where (i, j, k) are cyclic permutation of $\{1, 2, 3\}$. Then cohomogeneity one metrics with respect to G is described as

$$g = dt^2 + a(t)^2(\theta^1)^2 + b(t)^2(\theta^2)^2 + c(t)^2(\theta^3)^2. \quad (1)$$

For which $\{a(t), b(t), c(t)\}$ the resulting cohomogeneity one metrics are several metrics, for example, constant curvature metrics and product metrics. I will study for which $\{a(t), b(t), c(t)\}$ the resulting cohomogeneity one metric is Einstein or Ricci soliton.