Achievement

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I study the construction of Einstein structures and Ricci soliton structures on a given C^∞-differentiable manifold M^n of dimension n.

Let g_0 be a pseudo-Riemannian metric on M^n. If g_0 satisfies

$$2\mathrm{Ric}[g_0] + L_X g_0 + \alpha g_0 = 0,$$

where X is some vector field and α is some constant, then (M^n, g_0, X, α) is called *a Ricci soliton structure* and g_0 *a Ricci soliton.* Ricci solitons are special solutions of *the Ricci flow.*

I have two theme. The first is the following. From the result of Lauret(2003), the 3-dimensional Heisenberg group H_3 admit only one left-invariant Riemannian metric up to isometry and scaling. From the result of Baird and Danielo(2007) and Lott(2007), the left-invariant Riemannian metric on H_3 is a non-gradient expanding Ricci soliton. On the other hand, N.Rahmani and S.Rahmani (2006) proved that any left-invariant Lorentzian metric on H_3 is classified into three types g_1, g_2 and g_3, up to isometry and scaling. They showed that g_2 has negative constant curvature, g_3 is flat and g_1 is not Einstein. Under such the background, I characterize the left-invariant Lorentzian metric g_1 as a Lorentzian Ricci soliton. Moreover, I treated the group $E(2)$ of rigid motions of Euclidean 2-space and the group $E(1,1)$ of rigid motions of the Minkowski 2-space. In particularly, I proved that $E(2)$ has a non-flat Lorentzian Ricci soliton.

Next, the second is the following. Many researchers studied what kind of condition a cohomogeneity one metrics with respect to a G-action becomes Einstein metric and the Ricci soliton. I studied an extension of 3-dimensional unimodular Lie groups with a Ricci soliton structure, and when the evolution in the extra 1-dimension moves like the Ricci flow, then this has a Ricci-flat metric (an Einstein metric with zero Ricci tensor).

There exists a left-invariant coframe $\{\theta^i\}_{i=1}^3$ on three-dimensional unimodular Lie group G satisfying $d\theta^i = 2\theta^j \wedge \theta^k$, where (i,j,k) are cyclic permutation of $\{1,2,3\}$. Then cohomogeneity one metrics with respect to G is described as

$$g = dt^2 + a(t)^2(\theta^1)^2 + b(t)^2(\theta^2)^2 + c(t)^2(\theta^3)^2. \tag{1}$$

For which $\{a(t), b(t), c(t)\}$ the resulting cohomogeneity one metrics are several metrics, for example, constant curvature metrics and product metrics. When the triple of functions $\{a(t), b(t), c(t)\}$ satisfies the Ricci flow equation, I examined whether the resulting cohomogeneity one metric is a Ricci-flat metric. Moreover I proved that these metrics have the Hyper-kähler structure.