酒井 洋範

Research Plan

The theme of this research is relation between vortex invariants and (orbifold) Gromov-Witten invariants.

(Orbifold) Gromov-Witten invariants, quantum (orbifold) cohomology and quantum differential equation are important ingredients in algebraic geometry, symplectic geometry and mirror symmetry. On the other hand, the vortex invariant is an invariant which is defined by the moduli space of solution of symplectic vortex equation. The invariants have many application to symplectic geometry, e.g. Seiberg-Witten invariants and Gromov-Witten invariants. In particular, for a symplectic toric manifold the Gromov-Witten invariant coincides with the vortex invariant under a certain topological condition. This fact gives the ring structure of the quantum cohomology.

1. Vortex invariants and equivariant Floer theory

The topological condition is closely related to Givental's mirror theorem. The mirror theorem gives a relation between the J-function, which is defined by Gromov-Witten theory, and the I-function, which is defined by equivariant Floer theory. The vortex invariants must be directly related to equivariant Floer theory rather than Gromov-Witten theory. Thus in this research, we study direct relation between vortex invariant and equivariant Floer theory.

2. Vortex invariants of an orbifold and quantum cohomology

The result of accord of Gromov-Witten invariant and vortex invariant is expected to extend to orbifolds. For the extension, we have to extends vortex invariant to an orbifold. Moreover we need to construct the moduli space of solutions of symplectic vortex equation and to define vortex invariants. For the construction, it is important to study orbifolds as groupoids.

In this research, we construct vortex invariants of symplectic toric orbifolds and apply to quantum orbifold cohomology. In particular, we determine the ring structure of quantum orbifold cohomology. An efficient computation of orbifold Gromov-Witten invariants is another goal of this research. Moreover we expect a closed relation between the vortex invariants and equivariant Floer theory, which has not been rigorously defined yet.