

## The plan of our study

Every link can be realized as a closed braid. A. Kawauchi suggested the following project: enumerate the prime links by using the braid expressions and enumerate the closed connected orientable 3-manifolds by using the prime link table. Our main purpose is to make a table of the 3-manifolds according to this project.

We describe the outline of the project. A well-order (called a *canonical order*) was introduced on the set of links by A. Kawauchi [K] (see also A. Kawauchi and I. Tayama [KT1]).

We assign to every link a lattice point whose length is equal to the minimal crossing number on closed braid forms of the link and we call the number the *length* of the link. We note that a link  $L$  is smaller than a link  $L'$  in the canonical order if the length of  $L$  is smaller than that of  $L'$ , and for any natural number  $n$  there are only finitely many links with lengths up to  $n$ . Let  $\mathbf{L}^p$  be the set of prime links and  $\mathbf{M}$  the set of closed connected orientable 3-manifolds. Let  $\chi : \mathbf{L}^p \rightarrow \mathbf{M}$  be a map defined by  $\chi(L) = \chi(L, 0)$  (that means the result of the 0-surgery on  $S^3$  along  $L$ ). Then it is known that  $\chi$  is surjective and A. Kawauchi defined a map  $\alpha : \mathbf{M} \rightarrow \mathbf{L}^p$  by  $\alpha(M) = \min\{L \in \chi^{-1}(M) : L' \in \chi^{-1}(M), \pi_1(E(L)) = \pi_1(E(L')) \Rightarrow L < L'\}$  for  $M \in \mathbf{M}$ , where  $E(L)$  is the exterior of  $L$ . By using  $\alpha$ , we consider  $\mathbf{M}$  as a subset of  $\mathbf{L}^p$  and introduce the well-order into  $\mathbf{M}$ .

We completed the table of prime links with lengths up to 10, the table of prime link exteriors with lengths up to 10 and the table of closed connected orientable 3-manifolds with lengths up to 9. We will make our manifold table with lengths up to 10.

## References

- [K] A. Kawauchi, A tabulation of 3-manifolds via Dehn surgery, *Boletín de la Sociedad Matemática Mexicana* (3) 10 (2004), 279–304.
- [KT1] A. Kawauchi and I. Tayama, Enumerating prime links by a canonical order, *Journal of Knot Theory and Its Ramifications* Vol. 15, No. 2 (2006) 217–237