Plan for study

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I would like to sutdy various perturbation problems for the nonlinear elliptic partial differential equations, for example, nonlinear Schrödinger equations and nonlinear Schrödinger systems. In what follows, I give some topics of my study.

1. The existence and non-existence of positive solutions of the nonlinear Schrödinger equations for one dimentional case

We consider the following nonlinear Schrödinger equations for one dimentional case

$$-u'' + V(x)u = f(u) \quad \text{in } \mathbf{R}, \quad u(x) \to 0 \text{ as } |x| \to \infty \tag{$(*)_V$}$$

Setting $V^\lambda(x) = \lambda U(\lambda x) + 1$ for a positive function $U(x) > 0$, $V^\lambda(x)$ converges to a dirac function $a\delta$ ($a = ||U||_{L^1(\mathbf{R})}$) in distribution sense. Thus, letting u_λ be a least energy solution for $(*)_{V^\lambda}$, u_λ approaches in $C^1_{loc}(\mathbf{R} \setminus \{0\})$ to the non-torivial solutions of the equation

$$-u'' + a\delta u = |u|^{p-1}u \quad \text{in } \mathbf{R}, \quad u(x) \to 0 \text{ as } |x| \to \infty. \tag{3}$$

Moreover we can observe the following facts:

(i) If $|a| \geq 2$, then (3) does not have non-torivial solutions.
(ii) If $|a| < 2$, then (3) has an unique positive solution.

From the these facts, noting $a = ||U||_{L^1(\mathbf{R})} = ||V^\lambda||_{L^1(\mathbf{R})}$, we expect the followings:

(iii) When $||U||_{L^1(\mathbf{R})} \geq 2$, $(*)_{V^\lambda}$ does not have non-torivial solutions for large λ.
(iv) When $||U||_{L^1(\mathbf{R})} < 2$, $(*)_{V^\lambda}$ has at least a positive solution for large λ.

The existence of positive solutions of $(*)_V$ depends on the amount of integration of $V(x)$. In my plan, I would like to show (iii)–(iv) and I try to state the conditons for the existence and non-existence of positive solutions of $(*)_V$ by using the amount of integration of $V(x)$.

2. Unsolved problems in [3]

In [3], we construct solutions joining with a least energy solution of $(L)_{\Omega_1}$ and a solution of $(L)_{\Omega_2}$ which has large energy. We would like to construct solutions joining with large energy solutions of $(L)_{\Omega_1}$ and $(L)_{\Omega_2}$. Where, large energy solutions correspond to critical values which defind by using symmetric mountain pass theory.

One of the difficlties of this problem is a lack of the symmetries of the functional corresponding to $(*)_\lambda$. The functinonal corresponding to limitting equation has $\mathbf{Z}_2 \times \mathbf{Z}_2$ symmetry (i.e $I(u,v) = I(\pm u, \pm v)$ for all u, v). But the original problem $(*)_\lambda$ has only the $\mathbf{Z}_2$ symmetry (i.e $I(u,v) = I(-u,-v)$ for all u, v). To overcome this difficulty, we consider the following nonlinear elliptic systems which has similar structure.

3. Multiplicities of solutions of the nonlinear Schrödinger systems

I consider the following nonlinear Schrödinger systems:

$$\begin{aligned} -\Delta u + u &= u^3 + \epsilon F_u(u,v) \quad \text{in } \mathbf{R}^3, \\ -\Delta v + v &= v^3 + \epsilon F_v(u,v) \quad \text{in } \mathbf{R}^3. \end{aligned} \tag{$(4)_\epsilon$}$$

Here typical example is $F(u,v) = u^2v^2$ and $\epsilon \in \mathbf{R}$ is a parameter. When ϵ goes to 0, the solutions of $(4)_\epsilon$ approaches to a solutions of

$$-\Delta u + u = u^3 \quad \text{in } \mathbf{R}^3, \tag{5}$$

$$-\Delta v + v = v^3 \quad \text{in } \mathbf{R}^3. \tag{6}$$

By a standard symmetric mountain pass theory, (5) and (6) has infinite critical values, respectively. In my plan, I would like to show construct solutions correspond to pair of solutions for (5) and (6). Now regarding (5) and (6) as systems, corresponding functional has a $\mathbf{Z}_2 \times \mathbf{Z}_2$ symmetry. Thus this situations is similar to [3].