Research summary

[1] We can consider a invariant flat projective structure and flat affine structures on homogeneous spaces. Then the problem is to determine which homogeneous space admits this geometric structure. In this article the real Lie groups of dimension less than 6 are investigated concerning this problem. The result is the following: let L be a real Lie group of dimension ≤ 5. Then any L admits a invariant flat projective structure. On the other hand L admits a invariant flat affine structure iff $[Lie(L), Lie(L)] \neq Lie(L)$.

[2] This article is about the existence problem about invariant flat complex projective structures on complex Lie groups. I proved the correspondence between the set of invariant flat complex projective structures on complex Lie groups and the set of infinitesimal prehomogeneous vector spaces (abbrev. PVs) over the complex field. By using this correspondence and a classification of PVs given by Sato and Kimura I classified complex Lie groups admitting irreducible invariant flat complex projective structures.

[3] In this paper I investigated the existence problem of flat projective structures on manifolds and (flat) Grassmannian structures. A flat Grassmannian structure of type (n, m) on a manifold M is a maximal atlas $\{(U_\alpha, \phi_\alpha)\}_{\alpha \in A}$ of M such that ϕ_α takes values in the Grassmannian manifold $Gr_{m,m+n}$ and the coordinate changes $\phi_\beta \circ \phi_\alpha^{-1}$ belong to the projective linear group $PGL(m+n)$. A manifold equipped with a flat Grassmannian structure of type (n, m) induces a $GL(n) \otimes GL(m)$-structure of M.

In particular I constructed from a given manifold M equipped with a flat Grassmannian structure of type (n, m) an infinite sequence of projectively flat manifolds by successive castling transformations. Each projectively flat manifold is a principal fiber bundle over M with group $\prod_{i=1}^{j} PL(k_i)$. Moreover there is a bijective correspondence between the set of structure groups $\prod_{i=1}^{j} PL(k_i)$ of these constructed projectively flat manifolds and the set of certain solutions of a Grassmannian type quadratic equation.

I also described the relation of the base spaces in the above sequence. For instance the condition $m = 1$ corresponds to the assumption M admits a flat projective structure. Then castling transformation of M, we obtain the projective frame bundle M_n of M and the iteration yields a projective frame bundle $M_{n\times n^2+n-1}$ of M_n. The group action of $PGL(n)$ on M_n induces the action on $M_{n\times n^2+n-1}$ by the differential and the quotient manifold $M_{n\times n^2+n-1}/PGL(n)$ admits a flat Grassmannian structure. Furthermore M_{n^2+n-1} admits a $GL(n) \otimes GL(n^3 + 2n^2 - n - 1)$-structure $P_t M_{n^2+n-1}$ and the quotient $M_{n^2+n-1\times n^3+2n^2-n-1} := P_t M_{n^2+n-1}/GL(n) \otimes GL(1)$ is equipped with a flat projective structure.