Research Statement (summary)

Shintarô Kuroki

The numbers appearing below correspond to those in "List of achievements".

Extended actions of torus actions on torus manifolds. A torus manifold is a $2n$-dimensional T^n-manifold with fixed points, e.g. the complex projective space $\mathbb{C}P$ or even dimensional sphere. This manifold is defined by Hattori-Masuda as an ultimate generalization of toric manifolds from topological point of view. This is now regarded as one of the central objects in Toric Topology by several mathematicians.

It is important to find "most" natural group actions on manifolds. A study of extended actions on torus manifolds is related to this problem. In (14), I characterize all of such extended actions with Masuda. We define a root systems on the torus graph which is a combinatorial object induced from the torus manifold, and proved that the type of such groups is A, B or D only.

Cohomological rigidity problems for several classes of manifolds. Cohomological rigidity problem is the problem which asks whether cohomology ring isomorphisms induce homeomorphisms on manifolds. For general manifolds, of course the answer is NO. However, for toric manifolds this problem is still open.

In (10), I and Choi classify topological types of some class $\mathcal{M}$ in the torus manifolds appeared in (6), (9). We make a list of topological types of $\mathcal{M}$ by using cohomology rings and real characteristic classes. As a conclusion of this result, we give the subclass of $\mathcal{M}$ which satisfies the cohomological rigidity, and give a counter example of cohomological rigidity problems of torus manifolds proposed by Masuda-Suh.

A tori hyperKähler manifold (toric HK manifold for short) is the hyperKähler analogue of toric manifolds. This is a $4n$-dimensional space with T^n-action, e.g., cotangent bundle $T^*\mathbb{C}P$. I prove in (11) that equivariant cohomology distinguishes hyperhamiltonian structures on toric HK manifolds (this may be called equivariant cohomological rigidity).

A small cover is the real analogue of toric manifolds. This is an n-dimensional $\mathbb{Z}_2^n$-manifold, e.g., $\mathbb{R}P^n$. In (12), I and Masuda and Yu prove that aspherical small covers with virtually solvable fundamental groups are real Bott manifolds.

In (13), I and Suh introduces $\mathbb{C}P$-towers defined by iterations of $\mathbb{C}P$-bundles. For example, generalized Bott manifolds, flag manifolds of type A and C and Milnor hypersurfaces have such structures. We prove cohomological rigidity of $\mathbb{C}P$-towers up to 6-dimension, and give counter examples in 8-dimensional cases.

In (15), I prove that the simply connected torus manifolds with $H^2 = H^{odd} = 0$ is homeomorphic to S^8 or $\#S^4 \times S^4$. In (16), I characterize the equivariantly formal 6-dim torus manifold by using connected sum and gluing up to equivariant diffeomorphism.

GKM graph and gaph equivariant cohomology. A GKM manifold is a T^ℓ-manifold M^{2m} $(m \geq \ell)$ such that the orbit space of less than one dimensional orbits has the structure of a graph. For example, manifolds appearing above (except $\mathbb{C}P$-towers) are GKM manifolds. A GKM graph is defined by the graph, obtained by the orbit space of less than one dimensional orbits, labeled by tangential representations.

In (17), I define the class of GKM graphs called hypertorus graph, which contain the GKM graphs induced from the toric HK manifolds, and compute their graph equivariant cohomology rings. As a corollary, we have equivariant cohomology rings for wider class than the class of toric HK manifolds, e.g., manifolds defined in (32). In (31), I study combinatorial structures of some GKM graphs induced from GKM manifolds with $SU(n)$-extended actions. In particular, if there exists some class which may be called root systems of type A in graph equivariant cohomology, then their GKM graphs have the projection to the complete graph or after blowing up then we obtain such GKM graphs. From geometric point of view, these two graphs correspond to non-primitive manifold, i.e., manifolds isomorphic to $SU(n) \times_H N$, or primitive manifold, respectively.