Study proposal

Problem 1 Let S be a weighted $K3$ surface with at most ADE singularities or degenerated.

(1) Compute the automorphism group $\mathrm{Aut}(S)$ of S, and then compare $\mathrm{Aut}(S)$ with $\mathrm{Aut}(\tilde{S})$ of the minimal model $\tilde{S}$ of S if it exists.

(2) Consider how $\mathrm{Aut}(S)$ differs from $\mathrm{Aut}(\tilde{S})$.

Let $S_a \subset \boldsymbol{P}(a), S_b \subset \boldsymbol{P}(b)$ be generic anticanonical members in different weighted projective spaces.

(3) If the Picard lattices $\mathrm{Pic}(S_a)$ and $\mathrm{Pic}(S_b)$ are isometric, consider whether or not $\mathrm{Aut}(S_a)$ and $\mathrm{Aut}(S_b)$ are isomorphic.

Problem 1 may require the computation of $\mathrm{Aut}(S), \mathrm{Aut}(\tilde{S})$, which can be done by a careful study of blow-ups of singularities on S. It would be expected that Problem 1 can be applied to the compactification problem of moduli space of $K3$ surfaces with large Picard numbers.

Problem 2 Generalise the following **Fact** [**Kobayashi**] for S being a bimodal singularity, or other types of singularities:

Fact [**Kobayashi**] Let $S = (F = 0)$ be a unimodal singularity and $S^T = (F^T = 0)$ be its dual in the sense of Arnold's strange duality. Denote by $\widetilde{S}, \widetilde{S^T}$ the compactifications of S, S^T in the weighted projective spaces $\boldsymbol{P}(a), \boldsymbol{P}(b)$, respectively, and Δ_F, Δ_{F^T} be the Newton polytopes of F, F^T, respectively. Denote by Δ_a (resp. Δ_b) the polar polytope of $\boldsymbol{P}(a)$ (resp. $\boldsymbol{P}(b)$). Then, for a reflexive polytope Δ with $\Delta_b^* \subset \Delta \subset \Delta_a$, the polar dual polytope Δ^* of Δ, which is also reflexive and satisfies $\Delta_a^* \subset \Delta^* \subset \Delta_b$, induces an equivariant type of singularity as of S. Moreover, the Picard lattice of subfamily $\mathcal{F}_\Delta \subset \mathcal{F}_a$ associated to Δ is isometric to the lattice $\mathrm{Pic}(\mathcal{F}_a)$.

Problem 2 may require the classification of the invertible polynomials which appear in the construction of Bergland-Hübsch mirror pairs. It would be expected that Problem 2 can be applied to homological mirror symmetry and the relation between the derived categories from singularities and existence of curves on $K3$ surfaces.

Problem 3 Let S be a smooth toric hypersurface. Can one describe the Picard group $\mathrm{Pic}(S)$ in terms of the polytope ?

Being classical, Problem 3 is partially solved for S being such as a general anticanonical member in a smooth toric Fano 3-fold or in a toric Fano 3-fold with terminal singularities. Although the $\boldsymbol{Q}$-Picard group $\mathrm{Pic}(S)_{\boldsymbol{Q}}$ is studied, it is generally difficult to find out generators of $\mathrm{Pic}(S)$. As in the case of S being a general anticanonical member in a smooth toric Fano 3-fold, it is expected that Problem 3 requires a careful study of divisors on toric hypersurfaces.

Thus, it is intended to understand $K3$ surfaces from the point of view of complex algebraic geometry by studying automorphisms and Picard lattices of $K3$ surfaces and roles of $K3$ surfaces in mathematical physics.