Future research plan Atsuhide Mori

I would like to research on contact topology and obtain results on the next three subjects which require rich experiences in studying low dimensional cases:

i) Essential three dimensionality in (almost) contact topology: The dichotomy between tightness and overtwistedness has given vitality to contact topology in three dimensional case. Honda pointed out that we can regard a tight contact structure as a morphism in a certain cathegory which is a variant of triangulated cathegory. Here an edge of a triangle is called a bypass. I have a string-like analogue of bypass in higher dimensional contact topology which is a nice fattening of three dimensional bypass. Using it, I am now preparing a paper concerning a generalization of the above dichotomy. On the other hand, Martínez Torres showed that a leafwise symplectic foliation has a tautly foliated three dimensional submanifold and is indeed a leafwise fattening of the three dimensional taut foliation in the case where the leafwise symplectic forms are the restrictions of a closed 2-form on the manifold to the leaves. I suspect that a certain almost contact structure (confoliation) in higher dimension has a reduction to three dimensional case.

ii) Contact embeddings and singularity theory: Martínez Torres, generalizing my result, constructed a contact immersion of a given closed contact $(2n+1)$-manifold $M^{2n+1}$ into the standard $(4n+1)$-sphere $S^{4n+1} \subset \mathbb{C}^{2n+1}$ such that the pull-back of the trivial open-book on $S^{4n+1}$ is a symplectic open-book on $M^{2n+1}$. We call such an immersion a spinning. An embedded contact spinning can be considered as a generalization of a closed braid. I will study on complex singularities, especially on surface singularities from this point of view. While a Milnor fibration looks only at the monodromy of the pull-back open-book on $M^{2n+1}$, my braid concerns the embedding-type of the contact submanifold $M^{2n+1} \subset S^{4n+1}$ (or $S^{4n+3}$).

iii) Submanifolds foliated by Legendrian submanifolds: A Legendrian submanifold $L$ of a contact $(4n+3)$-manifold is a $(2n+1)$-manifold. Since a neighbourhood of the 0-section of the cotangent bundle $T^*L$ is also embedded, we can perturb $L$ to obtain a contact submanifold $L'$ as long as $L$ admits a contact structure. (Thus $L$ can not control the contact nature of $L'$.) On the other hand, a certain 1-dimensional family of Legendrian submanifolds of a contact $(4n+1)$-manifold forms a foliation, and does control a family of contact structures which converges to the foliation. This phenomenon was first found by Bennequin. For most of my results somehow related to this phenomenon, I would like to further persuit it.