Plan of Research

Takeshi Oota

We have introduced the so-called β-deformed matrix models of Selberg type in the year 2010. These model have remarkable property that they are exactly calculable models at any finite size N of the matrix. In the calculation, properties of the Jack symmetric polynomials play important roles. Recently, we are studying the β-deformed matrix models of Selberg type and related models. We will continue the study of these models.

Matrix models have played important roles in the modern developments of gauge theories and strings. One example of these is given by the gauge theory/matrix model correspondence which arises in the large N limit.

Since the proposal of the Alday-Gaiotto-Tachikawa conjecture, this gauge theory/matrix model correspondence collects renewed interests. The β-deformed (or β-ensemble) matrix models are used as a bridge to understand the 4d gauge theory/2d conformal field theory correspondence. The original β-deformed matrix model is related to the $SU(2)$ gauge theory with $N_f = 4$ fundamental matters. Various extension of this model, such as a generalization to $SU(N)$, a q-deformation, etc., have been proposed.

One of our plans of research is to investigate the q-deformed matrix models and to make clear some of their properties. The q-deformation introduces an additional parameter in the model. On the matrix model side, it corresponds to replace the Jack polynomials (one parameter) with the Macdonald polynomials (two parameters). On the gauge theory side, it corresponds to the lift from four-dimensional theories to five-dimensional ones.

It will be interesting to study various limits of the q-deformed models and to consider their connection with a new kind of gauge theory/conformal field theory correspondence. Several extension of the gauge theory/conformal field theory correspondence have been done, such as to consider the gauge theory not on the flat 4d Euclidean space but on non-trivial spacetime backgrounds, or to consider the supersymmetric conformal field theories. Determination of corresponding matrix models to these cases is one of interesting subjects.

In addition to generalize the matrix models of Selberg type, there is an unsolved problem of the choice of expansion basis. In our first paper of the matrix model of Selberg type, we have shown that by using the Jack symmetric polynomials, we can calculate various important quantities of the model. But for the quantities which is identical to the expansion coefficients of the Nekrasov partition function, the Jack polynomials are not suitable expansion basis. Some inhomogeneous symmetric polynomials which we do not know its characterization, provide a good basis for the gauge theory/ matrix model correspondence. It would be quite interesting, within the matrix model standpoint, to make clear the characterization of the inhomogeneous polynomials closely connected with the Nekrasov expansion coefficients.