Research Result

1. Moduli spaces of semi-stable sheaves and derived category of coherent sheaves on algebraic surfaces

In 2008–2009, we attacked a **conjecture of S. Mukai** (proposed in 1980) on a property of **stables sheaves on abelian surfaces**. The conjecture claims that generic stable sheaves on abelian surfaces of Picard number 1 has a **decomposition by semi-homogeneous sheaves** under certain conditions on Chern classes of the sheaves. In the collaboration with Kōta Yoshioka (paper [7]), we proved this conjecture affirmatively. In the cases of principally polarized abelian surfaces of Picard number 1, we also give an explicit construction of birational morphism between moduli schemes and Our construction involves certain **arithmetic groups and quadratic forms**, which are related to the auto-equivalences of derived categories of coherent sheaves.

In 2011-2012, we have been studying **Bridgeland stability conditions** on abelian or K3 surfaces. In the collaboration with Hiroki Minamide and Kōta Yoshioka (preprint [1]) we constructed stability conditions, studied their behavior under Fourier-Mukai transforms, and determined wall-chamber structure of the space of stability conditions. Analysis of the wall-crossing behavior of stable-objects was also done. In the preprint [2], we proved the projectivity of the moduli scheme of Bridgeland stable objects. In the preprint [3], we re-examined our paper [7] from Bridgeland stability point of view, and found that the arithmetic groups are corresponded to the crossing on walls of special type.

2. Quantum algebras and Macdonald symmetric functions

This is a sequence of collaborations with Hidetoshi Awata, Boris Feigin, Ayumu Hoshino, Masahiro Kanai and Jun-ichi Shiraishi.

Macdonald symmetric function is a symmetric function with two parameters, generalizing Schur, Hall-Littlewood and Jack symmetric functions. One of its important property is that it is a joint eigen-function of commuting difference operators. In the paper [1], we introduced **Feigin-Odesskii algebra**, which is realized as a commutative subalgebra of polynomial spaces with shuffle product, and constructed free field realization of Macdonald difference operators. We also studied **Ding-Iohara-Miki quantum algebra** and the Fock representation from the point of view of Macdonald symmetric functions.

The paper [2] studied the relationship between Ding-Iohara-Miki algebra and deformed W-algebra. We reconstructed the free field realization of deformed W-algebra in the tensor of Fock representations of Ding-Iohara-Miki algebra.

In the paper [5], we studied **K-theoretic AGT conjecture and Ding-Iohara-Miki algebra**. We proposed several conjecture that K-theoretic Nekrasov partition functions can be realized in the tensor of Fock representations of Ding-Iohara-Miki algebras, and confirmed several special cases.

3. AGT conjecture

AGT conjecture claims that **Nekrasov partition function**, which is related to the moduli space of rank 2 framed coherent sheaves on projective surfaces, coincides with the **Virasoro conformal block**. This conjecture implies that W-algebra will appear in the symmetry of moduli spaces of sheaves. As for this conjecture, we studied

a. Presentation of Whittaker vector of Virasoro algebra via Jack symmetric functions.
b. Zamolodchikov-type recursion formula.
c. K-theoretic AGT conjecture.

As for a, a degenerated version of AGT conjecture concerns about Whittaker vector, a special element of completed Verma module of Virasoro algebra. We studied this vector using free field realization and Calogero-Sutherland Hamiltonian, and obtained an expression via Jack symmetric functions (paper [4]).

As for b, we proved Zamolodchikov-type recursion formula of norms of Whittaker vectors of Virasoro algebra (paper [6]). By this work, **the proof of AGT conjecture for $G = \mathrm{SU}(2)$ mass-less case was completed**.

As for c, K-theoretic AGT conjecture predicts that K-theoretic Nekrasov partition function coincides with the norm of Whittaker vector of deformed Virasoro algebra. We conjectured a Zamolodchikov-type recursion formula, and proved it for K-theoretic partition function (paper [3]).