Results

Physics related to matrix model

In 2009, Alday-Gaiotto-Tachikawa conjectured the relations between "two dimensional conformal field theories" and "instanton part of four dimensional $\mathcal{N} = 2$ super Yang Mills-theories [1]." This relation is called AGT conjecture and has been studying in various points. In 2010, H. Itoyama and T. Oota had made the dictionary among Penner type matrix model and these two theories [2]. By using their dictionary, H. Itoyama, T. Oota, and I calculated the scaling limit of the Penner type Matrix model and confirm that the result coincides with the corresponding Nekrasov's partition function [3]. H. Itoyama and I analyzed the matrix model by using topological expansion and obtain the correction term of Seiberg-Witten pre-potential [4].

It is known that XXX spin chain model corresponds to a gauge theory. K. Muneyuki, T. Tai, R. Yoshioka and I find that another gauge theory corresponds to XXX Gaudin model obtained by taking a classic limit of XXX spin chain model and discuss the correspondence relation between these two gauge theories [5].

Quantum point-like interaction

The point-like interaction represented by δ-function potential is understood as the self-adjoint extension in functional analysis mathematically [6]. There are various point-like interactions except but δ-function potential and it is known that each point-like interaction have different nature respectively [7].

I studied the influence of point-like interactions on the thermal nature of systems. I. Tsutsui, T. Ichikawa and I showed that this is equivalent to influence of van der Waals force on the equation of state for the system [8-9]. In [9], we researched Lieb-Liniger model [10] which is a one-dimensional many body quantum system and has the δ-function potential and obtained the equation of state of van der Waals. Lieb-Liniger model is realized in recent years using the optical lattice thus our result predicts phase transition of a one-dimensional cold atom system.

On Lieb-Liniger model, it is confirmed theoretically and experimentally that the limit of its coupling constant to $+\infty$ of its state can be connected to the limit to $-\infty$ (super-Tonks-Girardeau state) smoothly. From this, A. Tanaka, T. Cheon and I proposed a cycle where coupling constant increases from +0 adiabatically, passed the limit to -infinity via the limit to +infinity, and result in -0. We find that final state is not the same as initial state although final coupling constant is the same as initial one. In mathematical context, this phenomenon is holonomy caused by geometric property of parameter space of Lieb-Liniger model.

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