Plan

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1. The slice-ribbon conjecture -in terms of the mapping class group-

As I stated in Result, I constructed potential counterexamples to the slice-ribbon conjecture in the preprint [1]. On the other hand, I think that the slice-ribbon conjecture is true. Therefore I will prove that potential counterexamples to the slice-ribbon conjecture are indeed not counterexamples. That is, we prove the following:

"All slice knots K obtained by annulus twists are ribbon."

Concretely, we prove this by the following steps:

• The case where K is a fibered knot[1]

In this case, whether K is a ribbon knot or not is expected to be equivalent to **whether the monodromy associated to K extends to an element of the mapping class group of the handlebody** (see [CD]). I will study as follows:
(1) I describe how annulus twists change an element in the mapping class group of the handlebody. And
(2) I describe a 4-manifold by using Kirby calculus according to the information of the element in the mapping class group of the handlebody. And
(3) I will show that K is a ribbon knot by studying the 4-manifold.

• The case where K is not a fibered knot

Some covering space of the complement of any knot is a surface bundle. Using this property, I reduce the arguments to the case where K is a fibered knot and prove that K is a ribbon knot.

Finally, I will solve the slice-ribbon conjecture extending the above result.

2. A construction of Lefschetz. fibrations

A Lefschetz fibration is an additional structure on a smooth 4-manifold. A fundamental question on Lefschetz fibrations is whether a Lefschetz fibration determines the smooth structure.

Park and Yun constructed some Lefschetz fibrations on a certain smooth 4-manifold. In particular, in 2011, they constructed these Lefschetz fibrations using Kanenobu knots. A key property is that Kanenobu knots are obtained from a fibered knot by using the "Stallings twists" twice.

I will construct infinitely many Lefschetz fibrations on a certain 4-manifold as follows:
(1) I construct infinitely many knots which are obtained from a fibered knot by the Stallings twists twice and are also obtained from the fibered knot by the annulus twist.
(2) I construct Lefschetz fibrations using these knots by Park-Yun's construction.
(3) I prove that these Lefschetz fibrations are diffeomorphic (as smooth 4-manifold).
(4) Finally, we prove that these Lefschetz fibrations are mutually distinct using invariants derived from the quandle theory.

Reference [CG] A. Casson and C. Gordon, *A loop theorem for duality spaces and fibred ribbon knots*, Invent. math (1983).

[1] That is, the case when the complement of K is a surface bundle over S^1.