Research Plan

The applicant has been interested in and investigated the relation between the structure of solutions to superlinear elliptic equations and its domain. From now the applicant will generalize results on non-positive or non-radial solutions to

$$
\begin{cases}
\Delta_{\mathbf{S}} u + \lambda u + |u|^{p-1} u = 0 & \text{in } B_{\theta_0}, \\
u + \kappa \dfrac{\partial u}{\partial n} = 0 & \text{on } \partial B_{\theta_0},
\end{cases}
\tag{1}
$$

that is, we consider (1) for $N \geq 3$. Then the linearized problem around $u \equiv 0$ is

$$
\begin{cases}
\Delta_{\mathbf{S}^N} w + \lambda w = 0 & \text{in } B_{\theta_0}, \\
w + \kappa \dfrac{\partial w}{\partial n} = 0 & \text{on } \partial B_{\theta_0},
\end{cases}
\tag{2}
$$

and the multiplicity of eigenvalues to (2) is greater than 2 in general. Thus, the argument used in the case of $N = 2$ is not valid for $N \geq 3$, and we are required to investigate (2) with a more general point of view.

In addition the solutions to (1) found in the case of $N = 2$ are local bifurcating solutions, and global properties of those are not known yet. Hence we will investigate global bifurcating solutions to (1). As for the global bifurcation of superlinear elliptic equations, Professor Yasuhito Miyamoto in the University of Tokyo studied the bifurcation structure of

$$
\begin{cases}
\Delta u + \lambda(-u + u^p) = 0 & \text{in } B, \\
\dfrac{\partial u}{\partial n} = 0 & \text{on } B,
\end{cases}
$$

where B is the unit ball in $\mathbf{R}^N$ (e.g., [1]). We have planned to apply his method for (1). Professor Miyamoto is also interested in the structure of solutions to (1), and the applicant has discussed the theme with him. Moreover we plan to have joint works as for the theme.

On the other hand, to investigate superlinear elliptic equations defined on manifolds in more detail, the applicant has planned to study the problems defined on surfaces of revolution. Those are more general problems than (1), and, by the study of the generalized problems, the applicant expects to clarify the structure of solutions to superlinear elliptic equations from a comprehensive point of view.

The applicant thinks to discuss the study of nonlinear elliptic equations with Professor Futoshi Takahashi, who is the host researcher and has proved many results on nonlinear elliptic equations. The applicant expects to develop his studies by the discussion with Professor Takahashi. Moreover the applicant thinks to collaborate on the problem of elliptic equations with him.

References

[1] Y. Miyamoto, Structure of the positive radial solutions for the supercritical Neumann problem $\epsilon^2 \Delta u - u + u^p = 0$ in a ball, UTMS Preprint Series 2013.