Future research plan

I would like to research on contact topology and obtain results on the next four subjects which require rich experiences in studying low dimensional cases:

i) Bivector fields and quantizations: Poisson structures (i.e. leafwise symplectic non-regular foliations) and Jacobi structures (i.e. leafwise contact non-regular folaitions) have been studied by many authors in the contexts of geometric quantization and deformation quantization. However there is no results connecting two structures on the same (fixed) manifold in those fields. I would like to build my confoliation theory with a wide viewpoint and from many angles.

ii) Essential three dimensionality in (almost) contact topology: The dichotomy between tightness and overtwistedness has given vitality to contact topology in three dimensional case. Honda regard a tight contact structure as a morphism in a cathegory "triangulated" by bypasses. I have a string-like analogue of bypass in higher dimensional contact topology. Using it, I am trying to generalize the above dichotomy. On the other hand, Martínez Torres showed that a corank one Poisson structure has a tautly foliated three dimensional submanifold and is indeed a leafwise fattening of the three dimensional taut foliation in the case where ω is a closed 2-form on the manifold. I suspect that a certain almost contact structure (confoliation) in higher dimension has a reduction to three dimensional case.

iii) Contact embeddings and singularity theory: Martínez Torres, generalizing my result, constructed a contact immersion of a given closed contact (2n+1)-manifold M^{2n+1} into the standard (4n+1)-sphere $S^{4n+1} \subset \mathbb{C}^{2n+1}$ such that the pull-back of the trivial open-book on S^{4n+1} is a symplectic open-book on M^{2n+1} . We call such an immersion a spinning. An embedded contact spinning can be considered as a generalization of a closed braid. I will study on complex singularities, especially on surface singularities from this point of view. While a Milnor fibration looks only at the monodromy of the pull-back open-book on M^{2n+1} , my braid concerns the embedding-type of the contact submanifold $M^{2n+1} \subset S^{4n+1}$ (or S^{4n+3}).

iv) Submanifolds foliated by Legendrian submanifolds: A Legendrian submanifold L of a contact (4n + 3)-manifold is a (2n + 1)-manifold. Since a neighbourhood of the 0-section of the cotangent bundle T^*L is also embedded, we can perturb L to obtain a contact submanifold L' as long as L admits a contact structure. (Thus L can not control the contact nature of L'.) On the other hand, a certain 1-dimensional family of Legendrian submanifolds of a contact (4n + 1)-manifold forms a foliation, and does control a family of contact structures which converges to the foliation. This phenomenon was first found by Bennequin. For most of my results somehow related to this phenomenon, I would like to further persuit it.