Research outline

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An a(lmost)-c(ontact) structure on an oriented (2n + 1) -manifold  $M^{2n+1}$  is a pair ([ $\alpha$ ], [ $\omega$ ]) of conformal classes of 1- and 2-forms  $\alpha$ ,  $\omega$  with  $\alpha \wedge \omega^n > 0$ . For example, given a contact form  $\alpha$  with  $\alpha \wedge (d\alpha)^n > 0$ , we obtain an a-c structure ([ $\alpha$ ], [ $d\alpha$ ]). On the other hand, if  $\alpha$  defines a codimension one foliation along whose leaf  $\omega$  is closed (i.e.  $\alpha \wedge d\omega = 0$ ), ( $\mathcal{F}, \omega | T\mathcal{F}$ ) is a corank one Poisson structure, and ([ $\alpha$ ], [ $\omega$ ]) is an a-c structure. I generalized the notion of confoliation due to Eliashberg and Thurston into higher dimension as intermediary a-c structures between these two examples. Recently, motivated by an effort of Verjovsky et al., Mitsumatsu constructed a corank one Poisson structure on  $S^5$ . In [11], I constructed a path of my confoliations connecting the standard contact structure on  $S^5$  with Mitsumatsu's example. In [12], I constructed a corank one Poisson structure on  $S^4 \times S^1$  while  $S^4$  is not almost complex. In [13], I improve my confoliation in terms of bivector fields in order to move toward its quantization.

Seifert surfaces in  $J^1(1,1) \approx S^3 \setminus \{*\}$  satisfies Bennequin's inequality, and surfaces in a contact 3-manifold are smoothly approximated by 'convex' ones. I [10] constructed a Seifert hypersurface in  $J^1(2,1) \approx S^5 \setminus \{*\}$  which violates the inequality and is not approximated by a 'convex' hypersurface. Lutz modified the contact structure of  $J^1(1,1)$  into exotic one. Using geometry of Brieskorn 3-manifolds, I [9] generalized the Lutz modification into  $J^2(2,1)$ . I obtained a 'convex' Seifert hypersurface which violates the inequality and obstracts symplectic fillability.

In [4] and [13] I constructed a certain immersion of a given contact  $M^3$  to  $J^1(2, 1)$ by using approximately holomorphic geometry. This result has been generalized by Martínez Torres. In [8] I smoothly isotoped the standard  $S^3$  in  $J^1(2, 1) \approx$  $S^5 \setminus \{*\}$  so that the restricted contact structure converges to the Reeb foliation (by Legendrian submanifolds of  $S^5$ ); and then becomes to an exotic contact structure. I explained the non-analyticity of Reeb foliation by using toric geometry on  $S^5$ .

Thurston and Winkelnkemper constructed a contact structure on a given openbook 3-manifold. I [3] showed that it comes from a symplectic filling if the monodromy is 'positive(right-handed)'. Loi and Piergallini showed that a 3-manifold is diffeomorphic to the boundary of a Stein domain iff it admits a 'positive' openbook. These results are later unified in Giroux's one-to-one correspondance between contact structures and stable positive stabilizations of open-books. I also showed that any contact structure on  $M^3$  can be deformed into a spinnable foliation. This implies that the relative Thurston inequality holds for many foliations with Reeb components in contrast to the Eliashberg-Thurston theory. With collaborators, I obtained relevant results: See [7] for homological overtwistedness, [6] for Dehn fillings, and [5] for a generalization of Bennequin's isotopy lemma.

I also have a collaboration [1] with Fukui on (in)stability of certain foliations.