Summary of Research

I. Virtual knot theory

A knot is an embedding of a circle into a 3-dimensional manifold. When this manifold is the sphere, knots can be described combinatorially using Gauss diagrams. It is possible to study Gauss diagrams alone, forgetting the knots: this is called *virtual knot theory* – it properly contains the classical theory. In my Ph.D thesis, I have defined a general version of Gauss diagrams that depends on a group G endowed with a $\mathbf{Z}/2$-valued homomorphism w. Every other Gauss diagram theory in the literature is contained in this one. I showed that when G and w are suitably chosen, this version encodes faithfully the knots in a given thickened surface – ie. a 3-manifold endowed with a line bundle projection onto a surface.

II. Finite-type invariants

Besides encoding knots, Gauss diagrams can also encode Vassiliev knot invariants, a large class of invariants which conjecturally separates knots. A conjecture of M.Polyak predicts that a certain kind of Vassiliev invariants (those presented by "arrow diagram formulas") can be understood as the kernel of an easily computable map. I proved that conjecture in a general framework that covers the case of knot theory in thickened orientable surfaces. As a corollary, one gets an improvement of Grishanov and Vassiliev's theorem on planar chain invariants. This work led to a paper accepted by the Journal of Knot Theory and Applications.

III. Towards detection of closed braids

So far, no algorithm has been found to tell whether a knot in the solid torus is isotopic to a closed braid. I proposed a plan to fill this gap, by studying an appropriate class of loops that can be drawn on a Gauss diagram. The first step is achieved, and consists in a characterization of Gauss diagrams of closed braids in terms of these loops. It remains to understand what happens to the loops when Reidemeister moves are performed, and I have a few results in that direction.

IV. Invariants of triple homotopies

The space of all knots is an infinite dimensional stratified space, whose finite codimensional strata correspond to singular knots. This observation has been the beginning of Vassiliev invariants theory. While usual knot theory studies the components of the complementary of all the singular strata, it is interesting to add some strata and study the topology of the resulting space. In a joint work with T.Fiedler published in the Journal of Knot Theory and Applications, we considered the space of regular knots together with 2-codimensional strata corresponding to a certain kind of triple points. It was already known that this space is connected ("any knot may be unknotted using triple homotopies"). We showed that it is not simply connected, by exhibiting a non trivial 1-cocycle. We also found a new formula for the Casson invariant (the simplest Vassiliev invariant) using "triple unknottings".