"On the solvability and global structure of the solutions set of a nonlinear elliptic equation involving the Dirichlet energy"

I have been carrying out the variational analysis of several nonlinear elliptic problems. Recently, I'm especially interested in a nonlinear elliptic problem involving the Dirichlet energy and consequently, getting several interesting results. In my future works, I'm going to progress my research under the following questions.

1. How can the nonlocal coefficient induce multiple solutions of the problem?
2. On the solvability of the critical problem in high dimension

In my recent research, I found out some interesting effect of the nonlocal coefficient on the existence of the solutions. Thanks to the nonlocal coefficient, our problem is solvable even for the situations under which the corresponding semilinear problem is never solvable. Furthermore, it can induce multiple solutions. Why can the nonlocal coefficient help the solvability of the problem and induce the multiple solutions? Variationally, it is regarded as the consequence of the change of the energy structure of the associated functional. It induces the structural complexity enough the functional to admit multiple local minimum or maximum (or minimax) points. In **1**, I would show another answer for the question. In particular, I shall give the answer from the point of the view; "an equation with *the nonlocal coefficient*". To this end, one of the key tools is the scaling property of the equation. We can observe the relationship between our nonlocal problem and corresponding semilinear problem through an appropriate scaling procedure. Combining this observation and the global results of the solutions set of the associated semilinear problem, we will be able to understand the reason why the nonlocal coefficient can work positively for the solvability of the problem. Furthermore we can expect to obtain the global structure of the solutions set of our nonlocal problem using this new approach. To my best knowledge, there are no previous works in which the researchers get the global structure of solutions set.

In **2**, I would try to solve the critical problem in high dimension. Certain results for the critical problem in three or four dimension are obtained by several works

including mine. But there are not enough work to conclude the solvability of the critical problem in high dimension. The reason lies in the fact that the limiting problem in high dimension admits multiple positive solutions. It causes a crucial difficulty in getting the criterion for the compactness of PS sequences. Carrying out further analysis for the concentration compactness phenomena of PS sequences and utilizing the new approach mentioned above, I would solve the critical problem in high dimension.