"The variational analysis on several nonlinear elliptic problems"

I have been studied about the existence of solutions of several nonlinear elliptic problems via variational methods. In particular I considered the following two problems.

1. Existence of infinitely many solutions of nonlinear Neumann problems
2. On the positive solutions of a nonlinear elliptic problem involving the Dirichlet energy

In **1**, I investigated nonlinear elliptic problems with nonlinear Neumann boundary conditions which are extensively studied recently. In particular, I dealt with the situation in which the problem has two different nonlinearities, known as the concave convex nonlinearities. My main aim is to generalize the result by Garcia–Azorero-Peral-Rossi(2004). To the first, I considered the convex concave nonlinearities with indefinite coefficients. As a consequence, I got a sufficient condition on the coefficients to get infinitely many solutions of our problem (The list of my published papers [4]). After that, I succeeded in much more generalization of this result in the joint work with R. Kajikiya at Saga University. In this work, we introduced the new condition called the local superlinear-sublinear condition. We showed the existence of two sequences of solutions of the problem under this condition.

In **2**, I studied about positive solutions of a nonlinear elliptic problem with the Dirichlet energy which is also extensively investigated in recent years. This problem is, for example, considered as the stationary problem of the Kirchhoff type wave equation which describes the free vibrations of elastic strings. Not only that but also it relates to a nonlocal parabolic type problem introduced by Chipot-Lovat(1997). Recently, because of the variational structure of this nonlocal problem, many researchers have been trying to solve it using the variational methods. Among other things, I considered the problem involving the critical Sobolev exponent ([2][3] and the other one is submitted for publication). In this case, because of the lack of the compactness of the Sobolev embedding, which is one of the most important tools in the variational analysis of nonlinear elliptic problems, the problem becomes much more delicate and thus, more interesting. The main difficulty lies in proving the appropriate convergence

of Palais-Smale sequences, which are suitable sequences of approximate solutions. To the first, I tried to extend the results on the concentration compactness phenomena of PS sequences by P.L. Lions(1985) or Struwe(1984) so that it can be applicable to our nonlocal problem. As a consequence, it was clearly pointed out that the nonlocality of the equation is brought to the corresponding *limiting problem*. Moreover, utilizing the nonlocality of the limiting problem, I succeeded in showing that PS sequences admit the desired convergence under appropriate hypotheses. To my best knowledge, this is the first work in which the four dimensional critical problem, which has much more difficulties than those in typical three dimensional one, is dealt with.