Research results

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For coprime integers $p(>0)$ and q, the (p,q)-cable knot $K^{(p,q)}$ of a knot K is a knot in the boundary ∂N of a tubular neighborhood N of K which is homologous to $pl + qm$ in ∂N, where l and m are a longitude and a meridian of K, respectively. For a knot invariant I, the function mapping K to $I(K^{(p,q)})$ is also a knot invariant, which is called the (p,q)-cable version of I. In general, it is expected that the (p,q)-cable version contains more information of knots than the original invariant has. For example, the cable version of the Jones polynomial, that is, the colored Jones polynomial is related to the volume conjecture. We focus on the Γ-polynomial, which is contained in the both the HOMFLYPT and Kauffman polynomials like the Jones polynomial. One of our interests is what kind of information of knots the cable version of the Γ-polynomial has. We have the following two results:

- **The braid indices of Kanenobu knots**

It is known that the Kanenobu knots have the same HOMFLYPT polynomial. It is known that every knot is presented as a closed braid. The braid index of a knot is the minimum number of strings needed for the knot to be presented as a closed braid. We often use the MFW inequality to give a lower bound of the braid index of a knot, which depends on the HOMFLYPT polynomial. Therefore, it is not easy to determine the braid indices of the Kanenobu knots. As a result, we distinguish the Kanenobu knots completely by computing the $(2,q)$-cable versions of the Γ-polynomials of the Kanenobu knots and give sharper estimations of the braid indices of the Kanenobu knots (Publication list [1,3,4,6]).

- **The cable Γ-polynomials of mutant knots**

We call a possibly different knot obtained from a knot by an operation called mutation a mutant knot. The mutant knot has properties similar to the original knot. Therefore, it is known that many knot invariants are invariant under mutation. For example, the HOMFLYPT polynomial, the Kauffman polynomial, and their $(2,q)$-cable versions are invariant under mutation. On the other hand, it is known that the $(3,q)$-cable version of the HOMFLYPT polynomial distinguishes a mutant knot pair. As a result, we show the $(3,q)$-cable version of the Γ-polynomial is invariant under mutation (Publication list [2,5]).