RESEARCH PLAN

YURIKO UMEMOTO

1. Research on the arithmetic nature of the growth rates of hyperbolic Coxeter groups

(1) In my recent paper[5], I gave examples of infinite families of compact Coxeter polytopes in 4–dimensional hyperbolic space, whose associated reflection groups with natural generating sets (Coxeter system) have 2–Salem numbers as the growth rates. Following this result, I will study whether there exist infinite families of noncompact Coxeter polytopes of finite volume whose associated reflection groups have 2–Pisot numbers as growth rates. This approach is based on results by Cannon, Wagreich [1] and Parry [4] that all the growth rates of reflection groups with respect to compact Coxeter polytopes in 2– or 3–dimensional hyperbolic space are Salem numbers or quadratic units, and the result by Floyd [2] that all the growth rates of reflection groups with respect to noncompact Coxeter polygons of finite volume in 2–dimensional hyperbolic space are Pisot numbers. To summarize and I am interested in studying the case of higher dimensions.

(2) I will approach the conjecture by Kellerhals and Perren [3], which says that the growth functions of reflection groups with respect to compact Coxeter polytopes in n–dimensional hyperbolic space have $n/2$ poles (if n is even) and $(n-1)/2$ poles (if n is odd) in the open interval $(0, 1)$. Since it is already solved for the case $n = 2$ or 3, so that I will start to study the case $n = 4$.

2. Research on the numerical distribution of the growth rates of infinite Coxeter groups

(1) At the present time, the minimal growth rate among the compact (reps. noncocompact and cofinite) hyperbolic Coxeter group with natural generating set (Coxeter system) acting on 2– or 3–dimensional hyperbolic space is known. However, it is not clear about the growth functions and the growth rates for the other generating sets than Coxeter systems, and especially, I am interested in and would like to approach the numerical distribution of such growth rates.

References

[1] J. W. Cannon, P. Wagreich, Growth functions of surface groups, Math. Ann. **293** (1992), 239–257.

[2] W. J. Floyd, Growth of planar Coxeter groups, P.V. numbers, and Salem numbers, Math. Ann. **293** (1992), no. 3, 475-483.

[3] R. Kellerhals, G. Perren, On the growth of cocompact hyperbolic Coxeter groups, European J. Combin. **32** (2011), 1299–1316.

[4] W. Parry, Growth series of Coxeter groups and Salem numbers, J. Algebra **154** (1993), 406–415.

[5] Y. Umemoto, The growth function of Coxeter dominoes and 2–Salem numbers, to appear in Algebraic and Geometric Topology.