Research plan

Schubert calculus for Springer varieties

(In collaboration with Megumi Harada in McMaster University and Tatsuya Horiguchi in Osaka City University.)

Springer varieties are defined as algebraic subsets in the flag variety of general Lie type, and they are fundamental spaces for geometric representations for Weyl groups. The ring structure of the cohomology of the Springer variety is computed by Toshiyuki Tanisaki in Osaka City University. We aim to understand its ring structure in terms of restrictions of Schubert classes from the cohomology of the ambient flag variety. We first study which Schubert classes can be chosen to form an additive basis of the cohomology of the Springer variety. After that, we will compute the expansion coefficients for a product of two classes from this basis, especially for the cases that one of those two are of degree 2 whose expansion coefficients are well known for the ambient flag variety.

Cohomology of Hessenberg varieties and representations of symmetric groups

(In collaboration with Megumi Harada in McMaster University, Mikiya Masuda in Osaka City University, and Tatsuya Horiguchi in Osaka City University.)

There are several important algebraic subsets of the flag variety; Springer varieties in geometric representation theory, Peterson varieties in connection with quantum cohomology of the flag variety, and the toric varieties associated with Weyl chambers. The *Hessenberg varieties* provides us a unified way of describing these spaces in the flag variety. In type A_n, they are defined from a matrix and a function $[n] \to [n]$ satisfying certain properties. Especially, the one defined from a nilpotent matrix with a single Jordan block is called *regular nilpotent Hessenberg variety*, and the one defined from a diagonal matrix with distinct eigen values is called *regular semisimple Hessenberg variety*. Our aim is to establish a relation between these two types of Hessenberg varieties in terms of representations of symmetric group on the cohomology rings.

Toric manifolds associated with root systems

As a special case of the Hessenberg varieties above, we have the toric manifold $X(\Phi)$ associated withthe Weyl chambers of a root system Φ. This space admits many interesting aspects such as the representation of symmetric group on its cohomology studied by Stembridge and a description as the moduli spaces of stable (n + 1)-pointed chains of projective lines studied by Losev-Manin.

The purpose of this research project is to answer the following question: given root systems Φ_1 and Φ_1, if $X(\Phi_1)$ and $X(\Phi_2)$ are homotopic, are Φ_1 and Φ_2 isomorphic? The author already showed that, if the root systems are irreducible of odd rank, then this question is true.