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Research Result

I am studying Lusztig and Kashiwara's canonical/global crystal basis and related topics.

Quivers and crystal basis

Lusztig has constructed the canonical basis via the geometry of moduli space of quiver representations. By the result of Kashiwara and Saito, it is known that the canonical basis is strongly related to the representation theory of preprojective algebra via the characteristic variety of the canonical basis. In master thesis, I studied the explicit bijection between them in affine cases.

Quantum unipotent subgroup and dual canonical basis

Cluster algebras are introduced by Fomin and Zelevinsky and motivated by the total positivity and the (dual) canonical basis of quantum groups. They are commutative rings whose generators are defined recursively using mutations. In serial works by Geiss, Leclerc and Schröer, it is known that there is a natural cluster algebra structure on the coordinate ring of the unipotent subgroup which is associated to a Weyl group element w and the set of cluster monomials is contained in Lusztig's dual semicanoncial basis. In the original work by Fomin and Zelevinsky, it is expected that the set of cluster monomials is contained in (specialization) of Lusztig's dual canonical basis. In doctor thesis, I studied the algebra $\mathbf{U}_v^-(w)$, which is introduced by Lusztig, Levendorskii-Soibelmann, and Kac-De Concini-Procesi, and its integral form and showed that the algebra is compatible with the dual canonical basis. Furthermore, we formulated the conjecture which can be considered as a quantization of Geiss, Leclerc and Schröer's works.

Graded quiver varieties and quantum cluster algebras

Monoidal categorification is a categorification of cluster algebra using monoidal abelian categories and it gives a conceptual proof of the positivity conjecture at arbitrary seed. Monoidal categorificaiton of cluster algebras of finite type and bipartite type is known by Hernande-Leclerc and Nakajima. In a joint work with Fan Qin, we studied the graded quiver varieties which is adapted to acyclic quiver and constructed the monodical categorification of acyclic (quantum) cluster algebras. As an application, we proved the quantization conjecture for the Weyl group element c^2, where c is the acyclic Coxeter word.