Plan of Research

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Meridian system of handlebody-knot

In general, for a handlebody-knot H, its meridian system M is not unique. We would like to determine the unique meridian system of a handlebody-knot.

The Alexander polynomial is an invariant for a pair of (H, M). If the Alexander polynomial of (H, M) is non-trivial, then replacement of a meridian system acts Alexander polynomials of H as $SL(n, \mathbb{Z})$. Thus, the meridian system of H is determined by the Alexander polynomial of H. By ordering Alexander polynomials of H, we can choice the unique Alexander polynomial of H and the unique meridian system.

For a pair of a genus g handlebody-knot and its meridian system, the bouquet of g-loops G is uniquely determined as a spatial graph. Thus, we use invariants of spatial graphs for the classification problem of handlebody-knots. I would like realize G such that the crossing number of G is relatively low by choice a good order.

Expansion of invariant for handlebody-knot

We introduce the graph G_H as an invariant of handlebody-knot H whose Alexander polynomial is non-trivial. There is infinite many handlebody-knots whose Alexander polynomial is trivial and whose Alexander ideal is non-trivial. I want to expand the invariant for such handlebody-knots.

Now I study Gröbner basis. Gröbner basis is uniquely determined for the pair a ideal of polynomial ring and its term order. Thus, the generator of Alexander ideal is uniquely determined by this. However, for handlebody-knot, $SL(n, \mathbb{Z})$ acts this ideal. This is the problem to expand G_H.