Plan of Research

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The work of Alday-Gaiotto-Tachikawa in 2009 leads to a renewed attention to the connection between two-dimensional conformal field theories and four-dimensional $\mathcal{N} = 2$ supersymmetric gauge theories. The two-dimensional correlation function (conformal block) is identified with the instanton partition function of the gauge theory.

This "2d/4d connection" has been generalized to various directions. One of these generalizations is a correspondence between the "q-deformed" two-dimensional conformal field theories and the "q-lifted" five-dimensional supersymmetric gauge theories. The q-deformed 2d theories have q-Virasoro or q-W algebras, while for the q-lifted 5d theories, the 5-th direction is compactified on a circle S^1 whose radius is proportional to $\log q$.

We have demonstrated that various 2d/4d connections can be explained from the 2d/5d connection as a parent by taking an appropriate limit for the parameter q. The $q = 1$ limit leads to the original 2d/4d connection, while a root of unity limit of q leads to the connection between the two-dimensional parafermionic conformal field theory and the $\mathcal{N} = 2$ supersymmetric gauge theory on (A-type) Asymptotically Locally Euclidean (ALE) space.

In our recent work, we showed that by taking the root of unity limit of q, the parafermion algebra can be obtained from the q-Virasoro or q-W algebras. We will study this limit in more detail, and try to elucidate various properties of the 2d/4d connection.

Also, it has been suggested that the Cherednik algebra (the double affine Hecke algebra) plays important roles in these 2d/4d correspondence. Hence, the study of the root of unity limit of Cherednik algebra may lead to some new interesting results.

Furthermore, one of future theme of our study is the extension to the gauge theories on general ALE spaces or to quiver gauge theories. It is proposed an construction of associated "Yangian" for the quiver gauge theories. In the Schur-Weyl duality, the Yangian is connected with a Hecke algebra. Hence, an extension to these direction will deepen our understanding of the gauge/CFT/matrix model correspondence.

And, there is a subclass of two-dimensional conformal field theories, called Wess-Zumino-Novikov-Witten (WNW) models, which have the symmetry of the current algebra. The conformal block of the WZNW models obeys the Knizhnik-Zamolodchikov equations which are a system of partial differential equations. In the context of the 2d/4d connection, these current blocks may play an important roles. We will study the q-deformation of the current blocks and their root of unity limits.